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FOREWORD

This report, entitled "Computer Program for the Calculation of Three-Dimensional Configuration Factors," LR 18905, was prepared by the Lockheed-California Company under NASA Contract NAS 9-3349. The Configuration Factor Program utilized under this contract was originally developed by Lockheed in 1963.

Other reports prepared under this contract are:

- LR 18899 A Transient Heat Transfer and Thermodynamic Analysis of the Apollo Service Module Propulsion System - Final Report
- LR 18900 A Transient Heat Transfer and Thermodynamic Analysis of the Apollo Service Module Propulsion System - Summary Report
- LR 18901 An Introduction to Spacecraft Thermal Control
- LR 18902 Thermal Analyzer Computer Program for the Solution of General Heat Transfer Problems
- LR 18903 Thermal Analyzer Computer Program for the Solution of Fluid Storage and Pressurization Problems
- LR 18904 Computer Program for the Calculation of Incident Orbital Radian Heat Flux

This report was originally written as LR 16657 by Mr. E. R. Linneman of Lockheed's Thermodynamics Department, who developed the logic of the program. The programming was done by Mr. L. H. Michel of Lockheed's Computing Services Department. The present report was revised and updated by Messrs. H. R. Holmes, K. J. Kahn, and B. A. Nevelli, also of the Lockheed-California Company.

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SUMMARY

A computer program has been developed by the Lockheed-California Company for the computation of radiation configuration factors. The principal features of the program are:

1. It is designed to include the effects of interference by intermediate surfaces.
2. It will handle planes, spheres, ellipsoids, elliptic cones, elliptic paraboloids, elliptic cylinders, parabolic cylinders, hyperbolic cylinders, hyperboloids of one or two sheets, hyperbolic paraboloids, and cylinders composed of two parallel or two intersecting planes, in any combination as either surfaces or boundary surfaces.
3. It is also designed to offer a high degree of flexibility in the number of surfaces and boundaries, in the methods of specifying these surfaces in the program input, in the computations performed, and in the trade-off between computing time and accuracy.

The use of this program is described in detail and is illustrated by several example problems. The program is written as a FORTRAN IV overlay program for the IBM 7094, with modular subroutine construction with two links, which provides for the finest possible grid size consistent with core size and efficient program operation.

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I - INTRODUCTION

The computation of the radiation heat transfer between an area A_1 of surface S_1 and an area A_2 of surface S_2 requires a geometric configuration or shape factor, $F_{1,2}$. This shape factor is defined for the standard geometry Figure 1-1 by equation (1-1), below:

$$F_{1,2} = \frac{1}{\pi A_1} \int_{A_1} \int_{A_2} \frac{\cos \phi_1 \cos \phi_2}{S^2} dA_1 dA_2 \quad (1-1)$$

The purpose of developing the present computer program is to allow computation of both point-to-surface and surface-to-surface shape factors, including the effects of shading by interfering surfaces.

By using modern digital computers, such as the IBM 7094, a numerical integration of equation (1-1) over the areas A_1 and A_2 can be obtained, provided that the surface equations, and therefore the integrand, are known. Given specified types of surface equations such as linear, quadric, trigonometric, exponential, etc., a program defining the integrand of equation (1-1) in terms of the surface equations can be written. For the Lockheed-California Company computer program, it was assumed that all surfaces would be quadric surfaces, that is, surfaces defined by equations of the form given by equation (1-2).

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \quad (1-2)$$

It was also assumed that all interfering surfaces would be quadric surfaces. This assumption gives eleven different surface types:



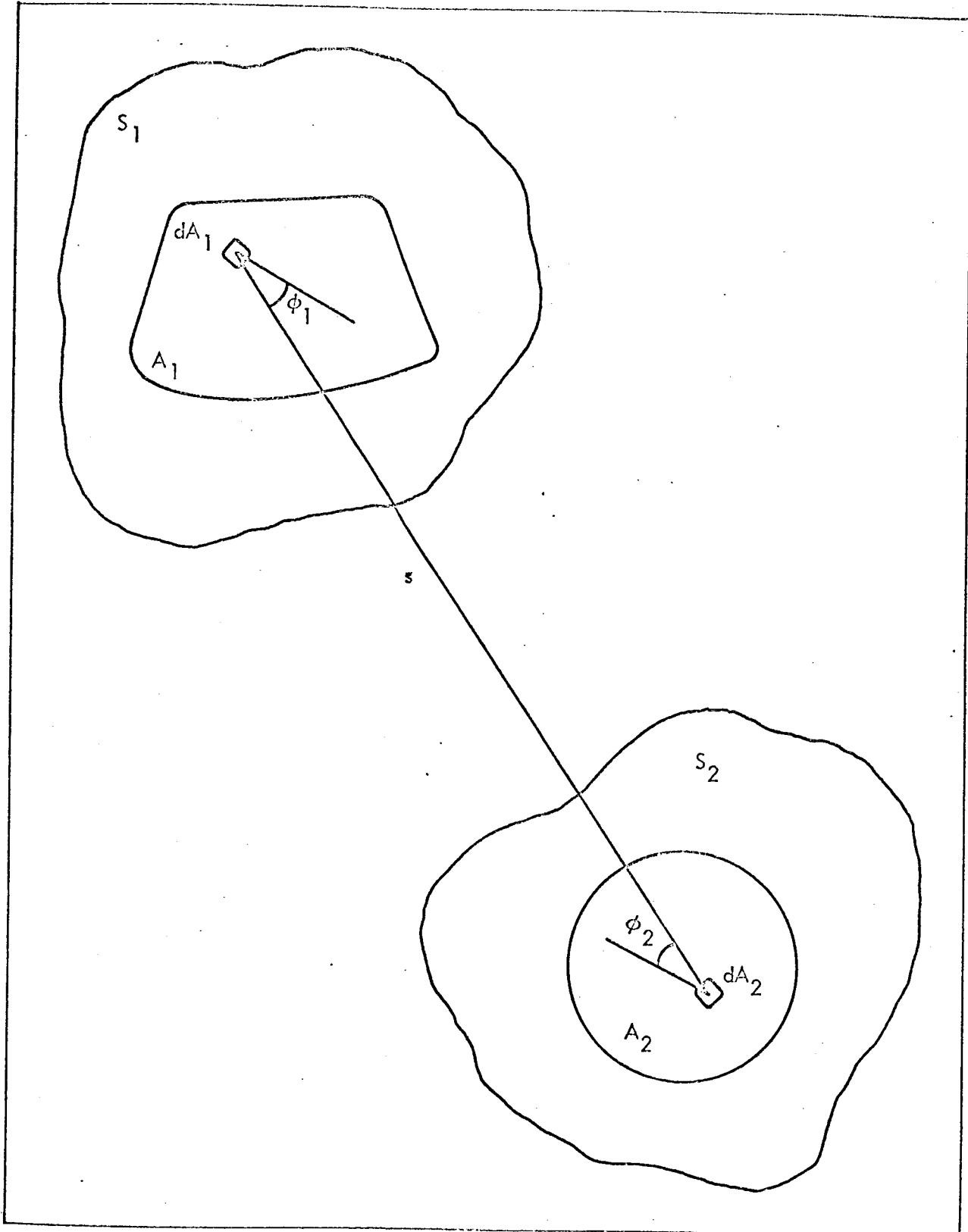


Figure 1-1. Standard Geometry

1. One plane
2. Two parallel or intersecting planes
3. Elliptic cylinders
4. Parabolic cylinders
5. Hyperbolic cylinders
6. Elliptic cones
7. Ellipsoids
8. Elliptic paraboloids
9. Hyperboloids of one sheet
10. Hyperboloids of two sheets
11. Hyperbolic paraboloids

Since a circle is a special case of an ellipse, the list includes spheres, right circular cones, and right circular cylinders, as well as the surfaces obtained by rotating ellipses, paraboloids or hyperboloids around an axis of the curve. Thus, 121 different pairs of distinct arbitrarily oriented surface types are available. Each of these surfaces can be given any boundary which can be defined by a series of segments, each of which is the intersection of the bounded surface with a quadric surface called the boundary surface.

Under these assumptions, the configuration factor program has been developed. The input routine will compute surface and boundary surface equations from a minimum of input data, thus facilitating the program input. The basic program is designed to solve equation (1-1) for the shape factor with or without interference, plus the areas of any given surface. A modification of this program permits the computation of an area weighting factor. This factor is defined as the percent of the total area of surface S_2 which is visible from an average point on surface S_1 .

II - PROGRAM DESCRIPTION AND CAPABILITIES

The program is written as a FORTRAN IV overlay program for the IBM 7094, with modular subroutine construction with two links, which provides for the finest possible grid size consistent with core size and efficient program operation.

LINK 1

Link 1 consists of input and grid point computation routines, and routines to find the quadric equation where necessary. Because the input for a large number of surfaces necessarily involves something of a bookkeeping problem, diagnostic routines have also been included. Up to 100 surfaces can be handled in one run.

Inputs may take the form of coefficients of the quadric equation (1-2), a set of points over the surface, coefficients of the equation of a simpler surface such as a plane or sphere, or as points on a simpler surface. Every open primary surface must be bounded by a boundary surface which is defined in a similar fashion. Using these data, the input routine chooses the set of equations which match the input format of each surface and boundary surface, and solves these equations for the coefficients of the quadric equation of the surface. This procedure is described in Appendix A. These coefficients are stored on tape for future reference. With these equations, the program can solve equation (1-1) for the required shape factors.

The program computes a set of grid points on the surface, given the dependent and two independent coordinates. If two independent coordinates of a grid point on a surface are given, equation (1-2) reduces to a quadratic equation in the dependent coordinate of the grid point. For example, if the independent coordinates x_o and y_o of a grid point are given, the dependent coordinate z can be computed by solving equation (2-1).

$$Cz^2 + (Ex_0 + Fy_0 + I) z + (Ax_0^2 + By_0^2 + Dx_0y_0 + Gx_0 + Hy_0 + J) = 0 \quad (2-1)$$

Similar equations are used if x or y are the dependent variables. The program uses a standard grid pattern to obtain the independent coordinates of the grid points. The dependent coordinate, the minimum and maximum values of each independent coordinate, and the number of grid points in one direction must be specified in the input.

The coordinates of the surface grid points, along with the previously computed coefficients of the surface and boundary equations, are stored on tape. This tape may be reserved from one run to the next so that once computed, the surface points may be used any number of times without recomputation.

LINK 2

Link 2 performs the actual integration, selecting surfaces from the reserve tape as called for by the input. Not all surfaces on the tape need be used, but any surface for which a shape factor is requested must be on the tape.

Interference by any surface may be checked. If the surface named does in fact interfere, the shape factor will be modified accordingly. A major aspect of the interference routine is that it requires only the equation of the interfering surface and its boundaries, thus reducing the core storage requirements.

For any pair of grid points, $P_1 = (x_1, y_1, z_1)$ on surface S_1 defined by the quadric equation $\theta_1(x, y, z) = 0$, and $P_2 = (x_2, y_2, z_2)$ on surface S_2 given by $\theta_2(x, y, z) = 0$, the integrand [equation (2-2)] of equation (1-1) can be computed.

$$\frac{\cos \phi_1 \cos \phi_2}{s^2} dA_1 dA_2 \quad (2-2)$$

The detailed derivation of the computer solution is given in Appendix B. Briefly, the procedure is as follows:

The integrand is given a numerical value for each pair of grid points, $P_1 (x_1, y_1, z_1)$ on area A_1 of surface S_1 , and $P_2 (x_2, y_2, z_2)$ on area



A_2 of surface S_2 . With this information, the program performs a numerical integration over the area A_2 and then over the area A_1 . The area A_1 is computed by a numerical integration of the quantities dA_1 over the area A_1 , and the shape factor is computed by equation (1-1). The program output gives the product of the area times the shape factor ($A_1 F_{1,2}$), the area on surface S_1 (A_1), and the shape factor ($F_{1,2}$).

The computation of the effects of shading is accomplished by means of a subroutine which operates during the computation of the integrand of equation (1-1). Before finding the integrand for a pair of grid points P_1 on surface S_1 and P_2 on surface S_2 , the subroutine finds the points in which the straight line through P_1 and P_2 intersects the shading surface. The points of intersection, if they exist, are then checked for location. If the line segment between P_1 and P_2 intersects the shading surface within its boundaries, the integrand is set equal to zero. If there are no points of intersection between P_1 and P_2 within the boundaries of the shading surface, the integrand is computed.

The machine time required to run this program depends largely upon the selected grid size and the number of interference surfaces. In general, the time for one shape factor computation ranges from 0.5 minutes to 2.0 minutes when run on the IBM 7094.



III - PROBLEM SET-UP

The basic program is designed to compute surface areas, surface-to-surface shape factors with or without shading, and point-to-surface shape factors with or without shading. Since the user may not have the equations of all surfaces and boundary surfaces, a routine has been provided to compute surface and boundary surface equations from a variety of input information. The program will automatically compute and print the equation of each surface and each boundary surface. All the surface area and shape factor computations, including the computation and storage of grid points for them, must be requested in the control section of the input.

The input for each quadric surface depends on how the surface is to be used. If the surface is to be used only as a shading surface and/or as the base of a point for the point-to-surface shape factor, surface and boundary equations, the side index, and the dependent variable are required. If the area of the surface is to be computed, or if the area is to be used in a shape factor computation, additional input is required. This includes a set of limits on the independent variables plus the number of grid points to be used so that a system of grid points can be defined on the surface.

The input for each problem is initiated by the selection of a general coordinate system. All equations, variables and other specifications are then referenced to these coordinates for the completion of that problem's input. A detailed description of these inputs follows.

SURFACE PARTITIONING AND SELECTION OF VARIABLES

For most practical engineering problems, the accuracy of the results depends largely on a technique called surface partitioning. Due to storage limitations, a single surface is divided into a finite grid size before numerical integration of equation (1-1) proceeds. This division is accomplished by a rectangular pattern of grid points projected onto the surface. Certain

surface orientations may therefore give distorted grids which may adversely affect the accuracy of the integration. To eliminate this problem, the user merely divides or partitions each surface into individual smaller surfaces which are then treated as separate cases by the program.

There are two fundamental reasons for surface partitioning. The first of these involves the selection of surface variables. For each surface input the user must specify the dependent variable. The dependent variable is defined as that axis of a selected fundamental coordinate system which is to be used in projecting a grid pattern on the surface in question. The two independent variables then define a plane in which the actual grid is constructed. Best accuracy therefore is obtained when this plane is most nearly parallel to the surface used. When closed or semi-closed surfaces are involved, no single axis of the chosen coordinate system will be close to perpendicular to the surface at all points and therefore serve as proper choice of the dependent variable. It is essential in these cases that the surface be partitioned - the extent of which depends on the time allotted for the solution. The angle between the dependent variable axis and any surface element must never be zero, nor should it be close to zero. If the angle is close to zero, erroneous shape factors may result.

The second purpose of partitioning is to allow finer grid patterns to be constructed. As the distance between adjacent grid points on any surface approaches the distance between the surfaces, computational accuracy drops considerably. Due to storage limitations, this is alleviated only by surface subdivision which reduces proportionally the grid point separation.

Once it has been decided what partitioning is required on a surface, boundary surfaces must be found which will subdivide the surface as selected. These are usually planes, or sets of planes, but may be any quadric surface. Examples of partitioning a cylinder and a pair of intersecting planes show typically how this is accomplished. Cylinders are divided by at least two planes, mutually perpendicular and intersecting along the axis of the cylinder. These are situated so that the chosen dependent variables intersect the resulting quadrants at their centerlines. Thus nowhere is the surface at an angle greater than 45° to the dependent variable. Two intersecting planes are partitioned for a different reason. For points on one plane near the



intersection line, distances to many grid points on the opposite plane will usually be small compared with grid point separations. This can only be eliminated by partitioning each plane into reasonably small rectangles — thereby reducing grid separations. Additional and more complete explanations of partitioning are included in Section VI, Example Problems.

Following this partitioning, the dependent variable to be used with each subsurface is identified with that surface. The third task involves the independent variables. For each of these variables the maximum and minimum values obtained on the surface are included in the input. These values define a rectangle in the plane of the independent variables bounded by their respective maximums and minimums. The program divides this rectangle into $(GS)^2$ equal subrectangles, where GS, the number of grid points, is specified as described below. The centers of the rectangles are then projected onto the surface, and those which fall on it are included in the integration over the surface.

The necessity for surface partitioning in most engineering applications cannot be overemphasized. If unreasonable results are obtained for a shape factor, surface partitioning is either incorrect or insufficient in nearly all cases.

GRID SIZE

Each surface that requires an integration also requires an input to describe the size of grid to be projected onto it. This input is termed the grid size (abbreviated GS), and is equivalent to the number of subdivisions between the maximum and minimum values of each independent variable. The maximum allowable value of GS is determined solely by computer storage requirements. It may have either of two maximum values, depending on the type of surface. For example, two grid points (one on top and one on the bottom) will be computed for a sphere, while only one grid point will be computed on a plane for each set of independent coordinates. An elliptic paraboloid, a hyperbolic paraboloid, or a parabolic cylinder may be oriented so that the axes of the parabolic cross-sections (in the case of the cylinder, the cross-sections perpendicular to the elements) are parallel to a coordinate axis. If so, these surfaces will only have one grid point per set of independent coordinates and



a maximum GS of 28 may be used. For all other curved surfaces or surfaces consisting of two planes, a maximum GS of 20 is used.

For many cases, such as a view factor to a simply shaped surface at a distance from the source, a GS less than the maximum should be used to save computer time. The maximum error is minimized however when the maximum allowable GS is used.

Note that computer time for the shape factor computation varies as the product $[GS(1) \times GS(2)]^2$, where GS(1) is the value of GS on the first surface and GS(2) is the value of GS on the second surface. Thus doubling GS on both surfaces increases computer time by a factor of 16. See example problem no. 3 for a specific comparison of grid size, accuracy, and computer time.

SIDE INDEX

The Side Index is an input flag which specifies the required side of any surface in question. The two sides of a surface are defined as positive and negative, the positive side being that one which faces a predetermined reference point for that surface.

The reference point method of side identification is based on the fact that no quadric surface can contain all of the points:

- | | |
|---------------|---------------|
| 1. (0, 0, 0) | 6. (0, -1, 0) |
| 2. (1, 0, 0) | 7. (0, 0, -1) |
| 3. (0, 1, 0) | 8. (1, 1, 0) |
| 4. (0, 0, 1) | 9. (1, 0, 1) |
| 5. (-1, 0, 0) | 10. (0, 1, 1) |

For each surface, the program tests these points in order. The first of these points which does not lie on the surface is defined as the surface reference point. The positive side of a surface (side index 1.0) is defined as that side which faces the reference point while the other side is called the negative side (side index -1.0). In order to input the required side index (+1.0 or -1.0), the user must independently determine the surface reference point.



SURFACE INPUT SPECIFICATION

This program is based on the assumption that each surface and boundary can be described by equation (3-1).

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \quad (3-1)$$

Each surface is identified with equation (3-1) by any one of ten methods or formats, designated by separate input flags. The ten surface input specifications, the required data, and the surfaces which can be described by each format are summarized in Table 3-1. The data required for each of the formats, Flags 0 through 7, are illustrated in Figures 3-1 through 3-8. The points labeled P represent points which will define a unique surface, while the points labeled Q represent an improper choice of points which will fail to define a unique quadric surface.

If the user has or can readily obtain the equation of the surface, the easiest and most accurate way to describe a surface is to use input Flag -1 and give the ten coefficients A through J of the equation. This is the most common way to describe any surface, and should be used in most cases.

If the equation is not easily obtainable or if a non-quadric surface must be approximated by a series of quadric surfaces, the input will depend on the type of surface and the information available to the program user. Eight different input formats are available with special ones for planes, spheres, cylinders and quadrics of revolution, as well as a general method which will handle any surface except a plane. In general, the lower numbered flags are least likely to result in errors (excepting Flag 8).

The first special format (Flag 0) is the only way, other than by the equation, to input a plane. This format requires three points on the plane which are not on a straight line.

There are two special formats for the sphere. The first (Flag 1) requires the coordinates of the center and the length of the radius. The second (Flag 2) requires the coordinates of four points on the sphere which are not all in the same plane.



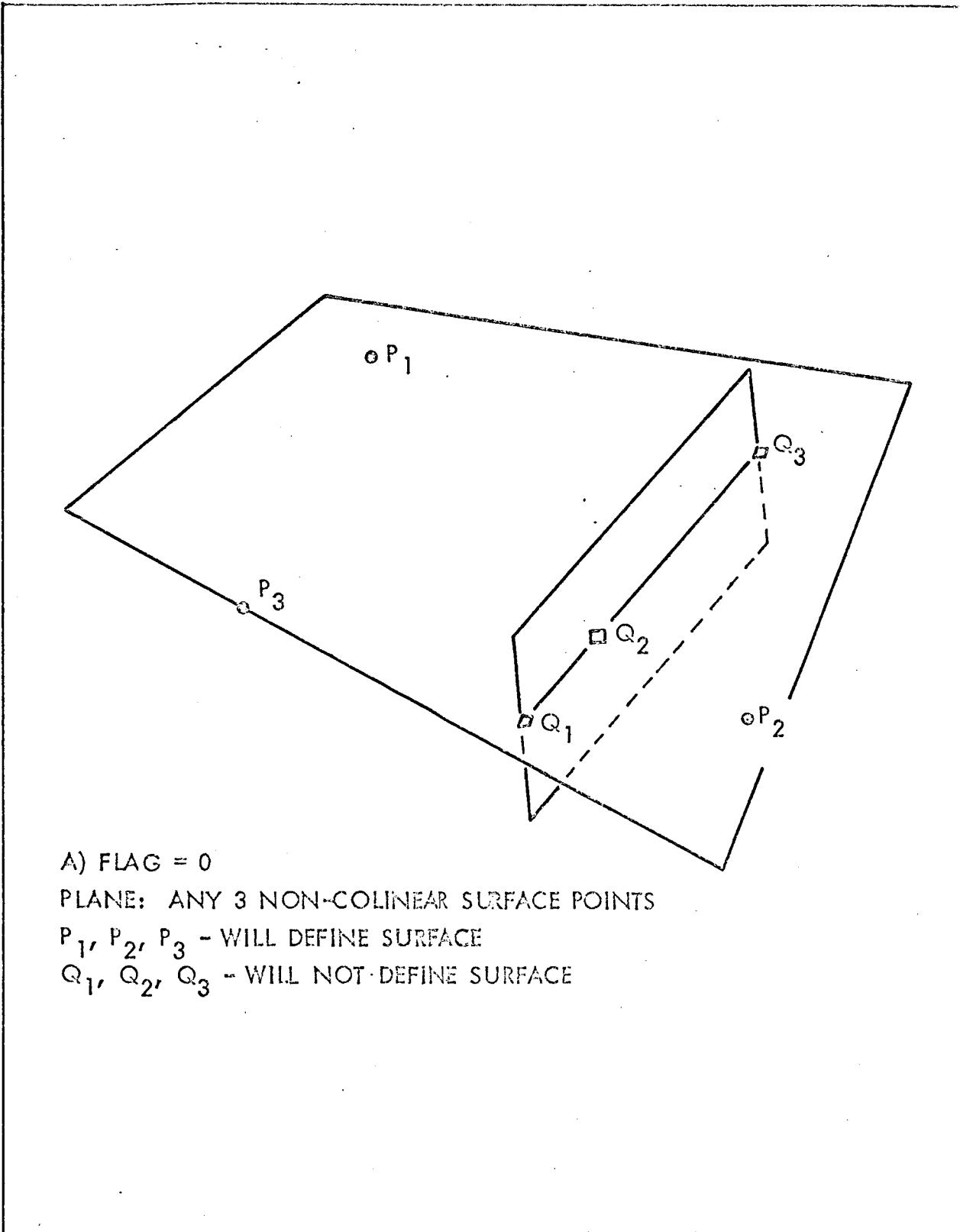


Figure 3-1. Surface Input - Plane

TABLE 3-1
SURFACE INPUT SPECIFICATION

Flag	Surface Description	Required Input Data
-1	Quadric surface	The 10 coefficients of the surface equation
0	Plane (Fig. 3-1)	3 non-colinear surface points (P_i , $i = 1, 2, 3$)
1	Sphere (Fig. 3-2)	Center (P_k) and radius (R)
2	Sphere (Fig. 3-3)	4 non-coplanar surface points (P_i , $i = 1, \dots, 4$)
3	Right circular cylinder (Fig. 3-4)	2 axis points (P_1 and P_2) plus radius (R)
4	Right circular cylinder (Fig. 3-5)	Direction numbers (l:m:n) of an element plus 3 points (P_i , $i = 1, 2, 3$), each on a different element
5	Quadric cylinder (Fig. 3-6)	Direction numbers (l:m:n) of an element plus 5 points (P_i , $i = 1, \dots, 5$), each on a different element
6	Quadric of revolution (Fig. 3-7)	2 axis points (P_0 & P_4) plus 3 surface points (P_i , $i = 1, 2, 3$), each on a different cross-section perpendicular to the axis
7	General quadric surface (Fig. 3-8)	9 spaced surface points (P_i , $i = 1, \dots, 9$)
8	Different side or section of preceding surface	Surface number of the preceding surface. The program will use the equation of the indicated surface.

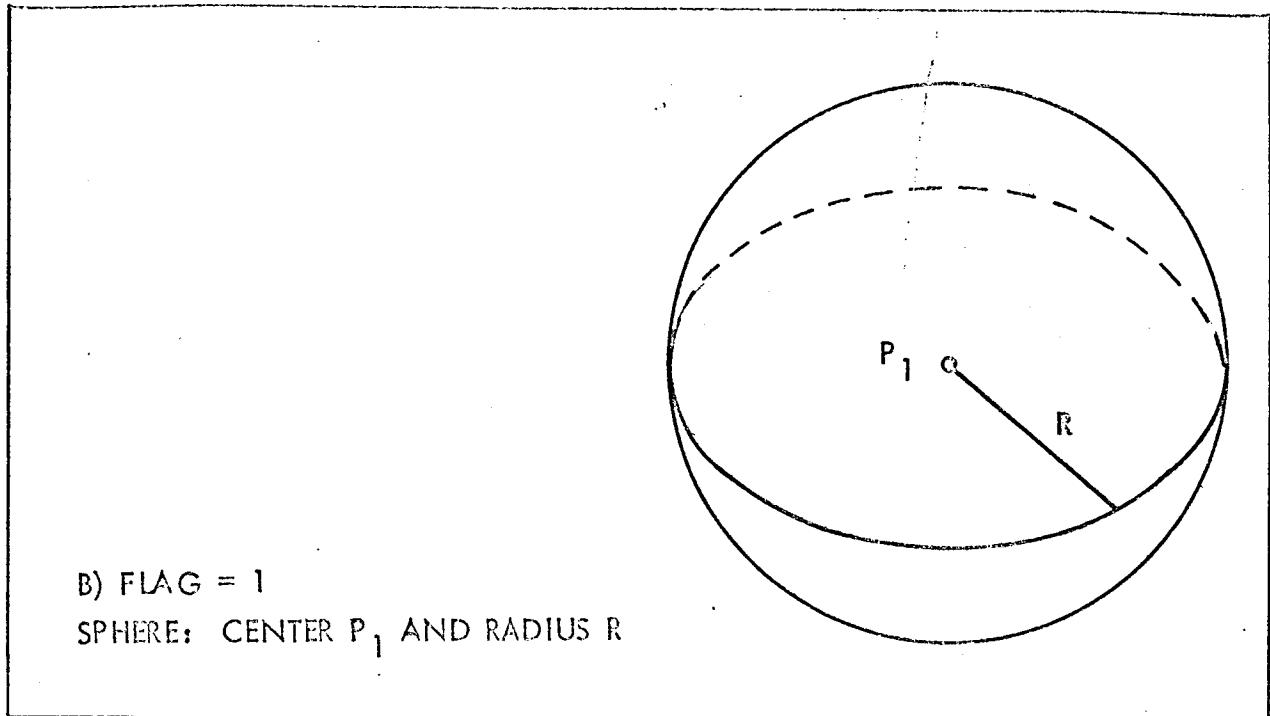


Figure 3-2. Surface Input - Sphere #1

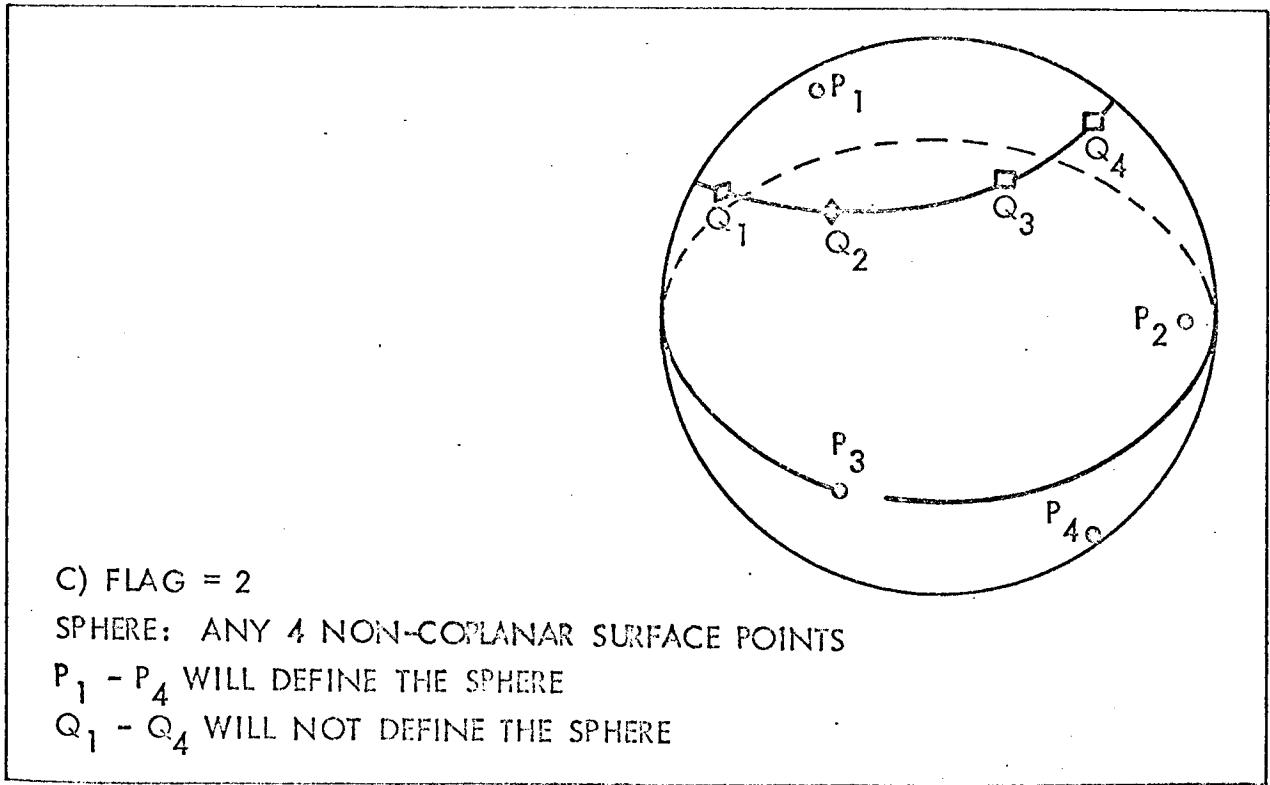
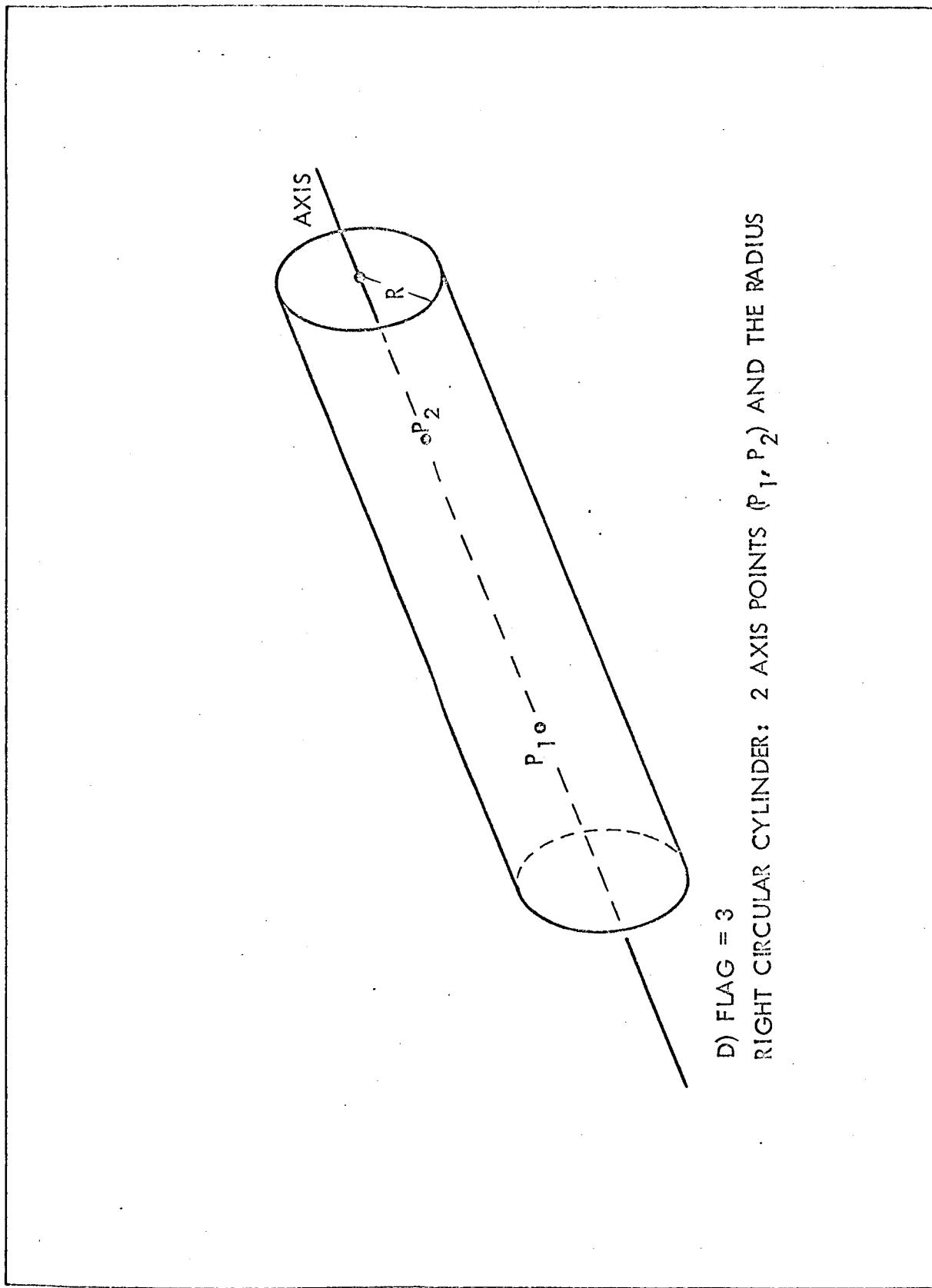


Figure 3-3. Surface Input - Sphere #2



D) FLAG = 3
RIGHT CIRCULAR CYLINDER: 2 AXIS POINTS (P_1, P_2) AND THE RADIUS

Figure 3-4. Surface Input - Right Circular Cylinder #1

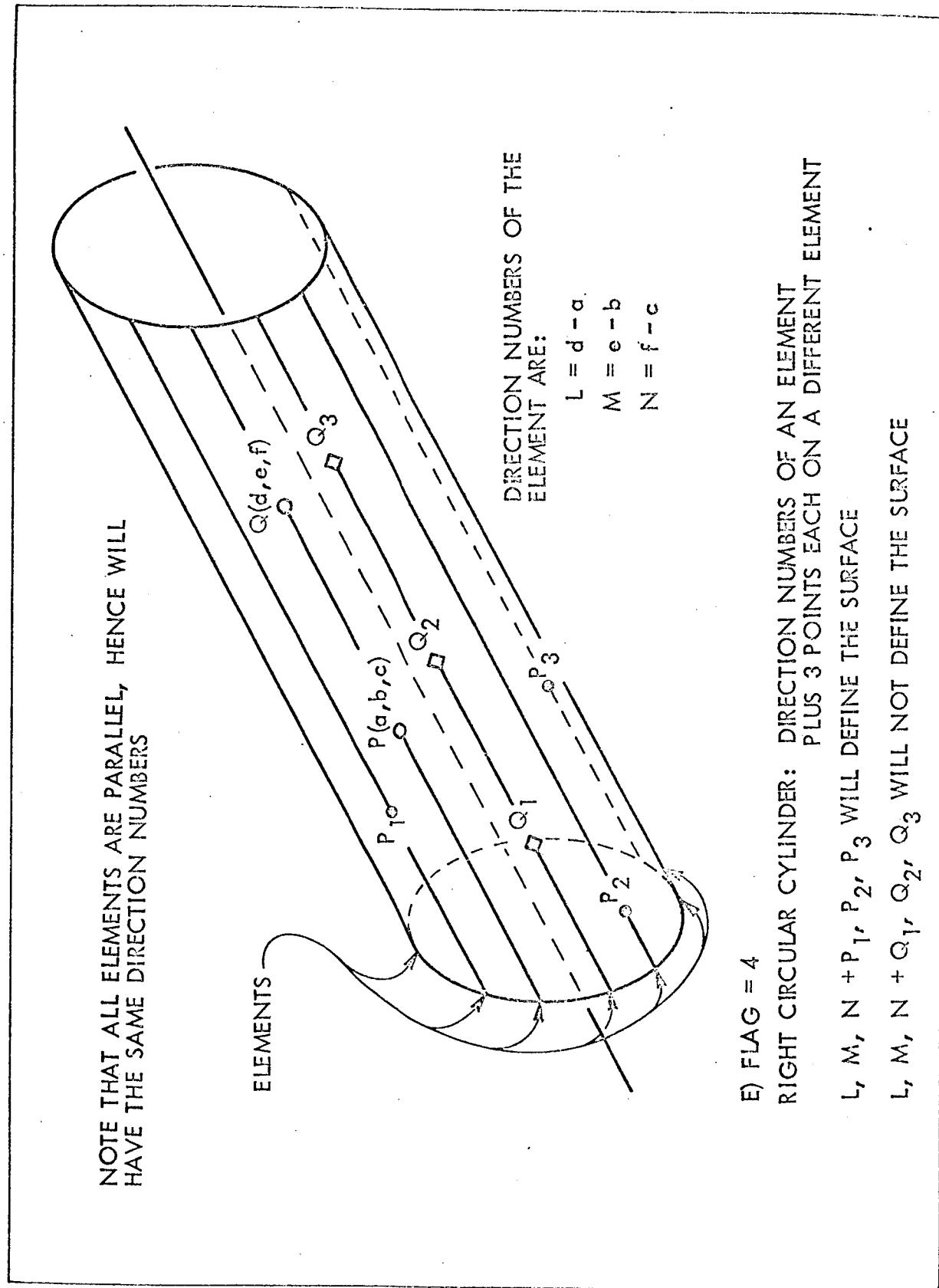
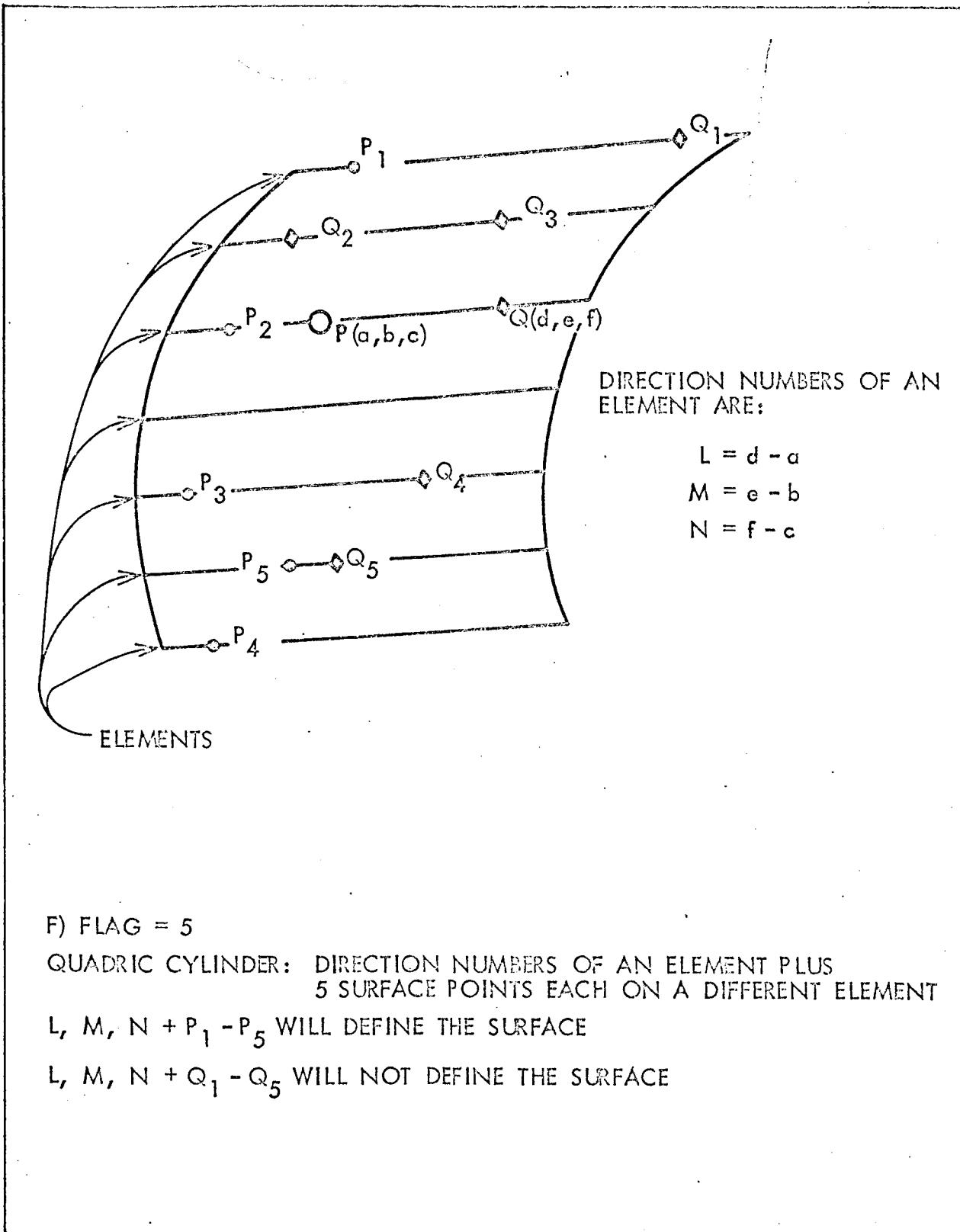


Figure 3-5. Surface Input - Right Circular Cylinder #2



F) FLAG = 5

QUADRIC CYLINDER: DIRECTION NUMBERS OF AN ELEMENT PLUS
5 SURFACE POINTS EACH ON A DIFFERENT ELEMENT

$L, M, N + P_1 - P_5$ WILL DEFINE THE SURFACE

$L, M, N + Q_1 - Q_5$ WILL NOT DEFINE THE SURFACE

Figure 3-6. Surface Input - Quadric Cylinder

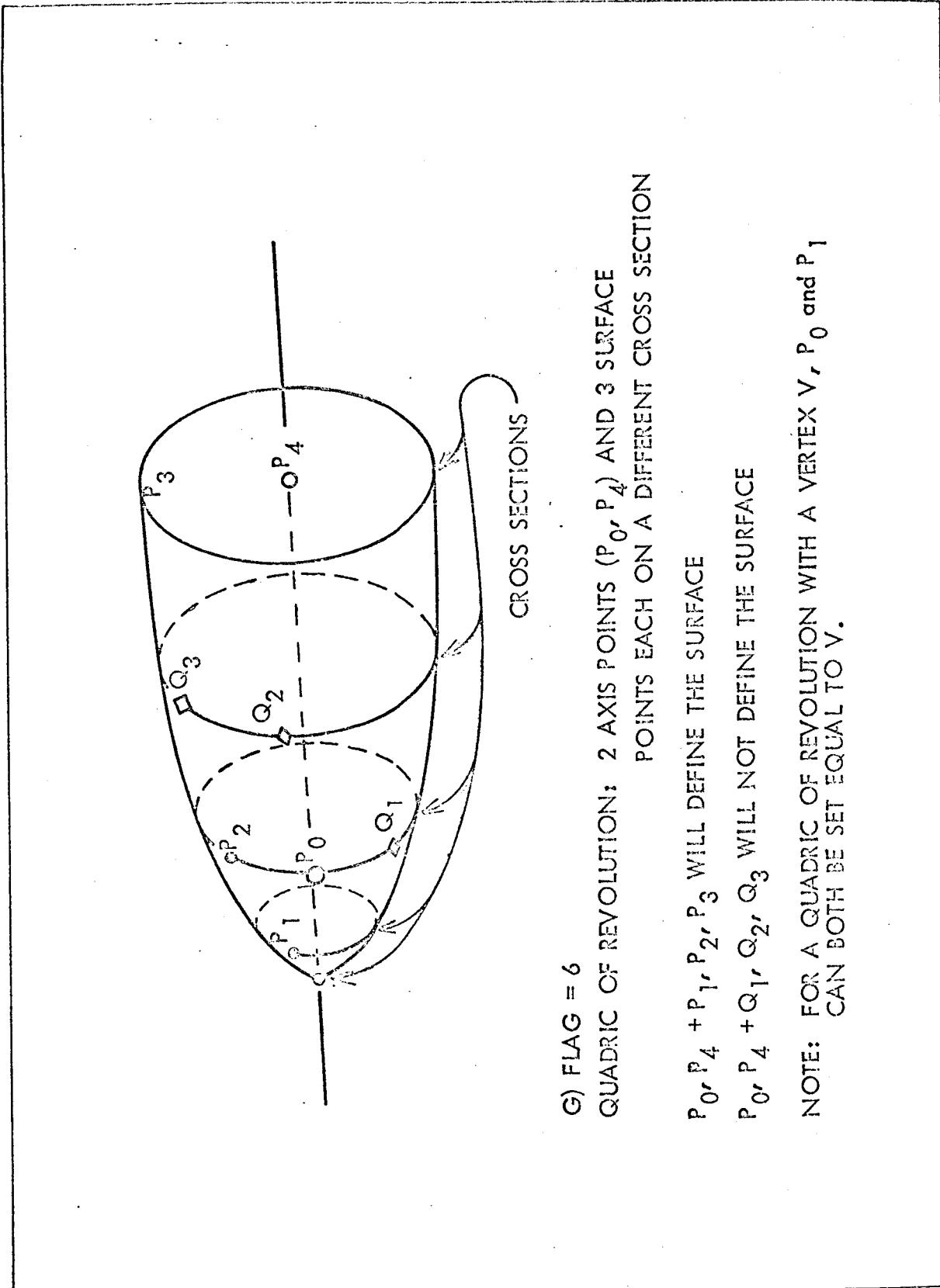
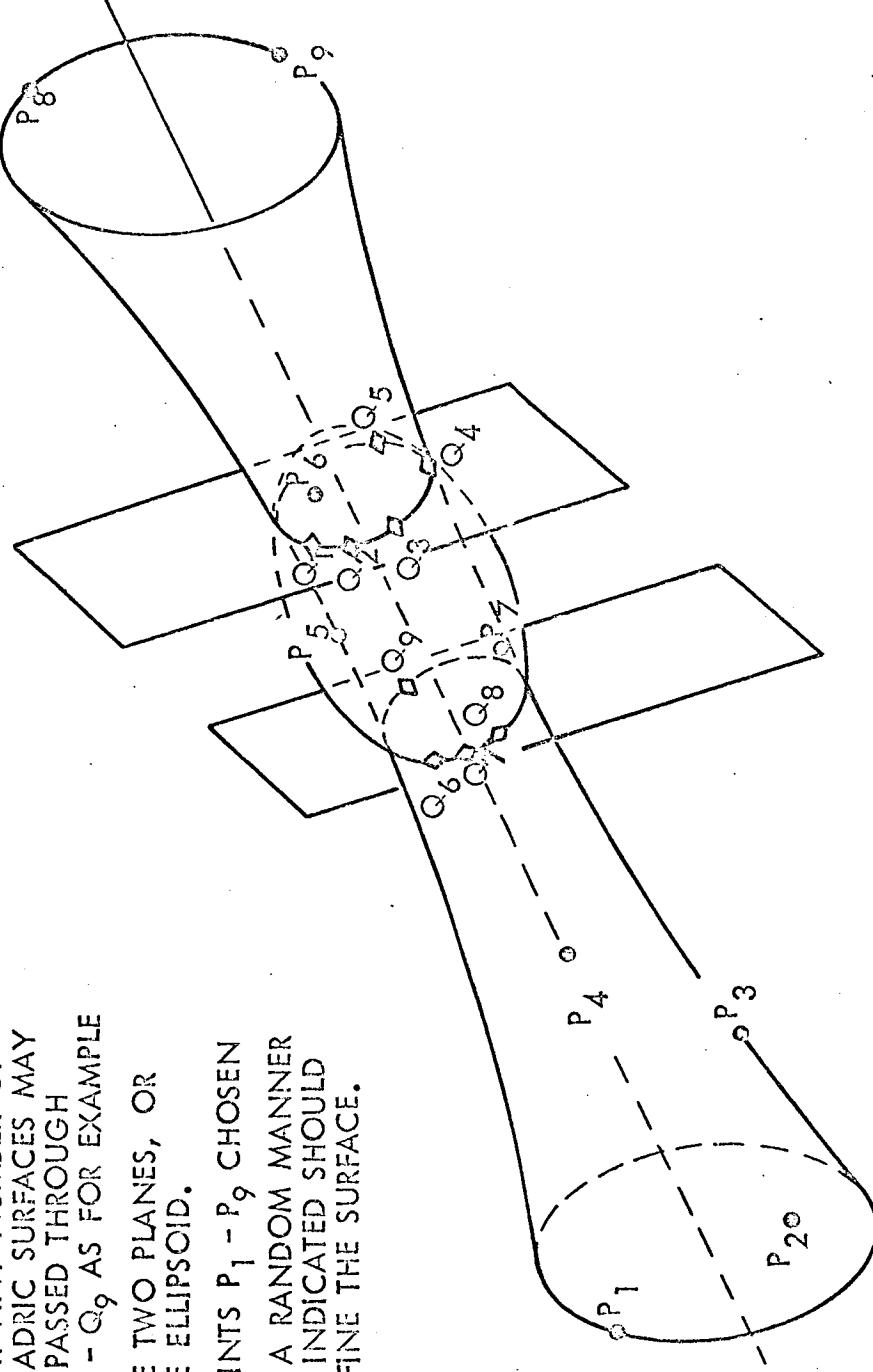


Figure 3-7. Surface Input - Quadratic of Revolution

NOTE: THAT ANY NUMBER OF QUADRATIC SURFACES MAY BE PASSED THROUGH POINTS $P_1 - P_9$ CHOSEN IN A RANDOM MANNER AS INDICATED SHOULD DEFINE THE SURFACE.



H) FLAG = 7
QUADRATIC SURFACE (NOT A PLANE): ANY NINE SURFACE POINTS WHICH DO NOT LIE ON ANY OTHER QUADRATIC SURFACE.

Figure 3-8. Surface Input - General Quadratic

The program includes three formats for quadric cylinders. The first two are for right circular cylinders only. The first (Flag 3) requires two points on the axis of the cylinder plus the radius, while the second (Flag 4) requires the direction numbers of an element of the cylinder plus three surface points, no two of which are on the same element. The third cylinder input (Flag 5) is for any arbitrary quadric cylinder. It requires the direction numbers of an element plus five surface points, no two of which are on the same element. In general, any cylinder can be defined as the collection of all the parallel straight lines satisfying some specified condition. Using this definition of a cylinder, an element of the cylinder will mean any of these straight lines. The direction numbers ($l; m; n$) of an element can be obtained by taking any two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on the same element, then $l = x_2 - x_1$, $m = y_2 - y_1$, and $n = z_2 - z_1$.

The last of the special input formats (Flag 6) is for a quadric of revolution other than a plane. This includes spheres, right circular cones, cylinders, ellipsoids, paraboloids, and hyperboloids of revolution. The required input is two axis points and three points on the surface with no two of the surface points on the same cross section perpendicular to the axis. Note that in the special case where the conic crosses the axis (i.e. all but the cylinders and hyperboloids of 1 sheet), the vertices may be used as surface points. Hence, the vertices can be used as both axis and a surface points for these surfaces.

The general quadric input (Flag 7) requires nine surface points. These points must be so located that there will be only one quadric surface which contains them. The best way of achieving this is to choose points which are uniformly distributed over the area of interest. It is essential that no more than five points lie in any one plane (a maximum of four is preferable whenever possible). Unless the surface consists of two parallel or intersecting planes, the points must not all lie in two planes. Since it is possible to choose points which do not define a unique quadric equation, the special input formats should be used whenever possible.

In addition to these surface input formats, there is an input format (Flag 8) which will duplicate a preceding surface equation in case a different area, side, or dependent variable is required on a quadric surface. For

example, to use a different hemisphere of a sphere, or the opposite side of a plane, a new surface must be defined and indexed. However, since the surface equation will be the same in both cases, it is more convenient and economical to duplicate the equation of the original surface than to recompute it. This input (Flag 8) requires only the surface number of the surface whose equation is to be duplicated.

In any case where the machine must compute a surface equation, the nature of the digital solution with its restriction to eight significant figures will limit the accuracy of the equation. In particular, coefficients which should be zero may be given a finite value. If the chosen points do not define a unique quadric surface, the program should compute all zero coefficients and write out "Surface (S) is not defined." However, with digital computation of the determinants one or more of these could yield non-zero coefficients. These coefficients would define the equation of a false surface. To counteract this possibility, each equation which is computed from surface points (input Flags 0, 2, 4, 5, 6, and 7) is sent through a test routine which evaluates the equation at each of the surface points. These values are printed out below the surface reference point q in the order in which the surface points appear in the input. If any one of these values exceeds 0.1, the surface is rejected. Since this is not a flexible test, the user should compare these values with the coefficients of the equation and reject any equation for which one or more of these values approaches the order of magnitude of the average coefficient.

BOUNDARY SURFACE INPUT SPECIFICATION

All boundaries are defined by the intersection of the quadric surface with a quadric boundary surface. The input for a boundary surface will therefore resemble the input for a surface. A boundary surface can be defined in eight ways, as summarized in Table 3-2.

The first (Flag -1) is to use any of the surfaces listed in the surface input as a boundary surface. The required input is the surface number. This gives an easily defined boundary if two surfaces intersect.

TABLE 3-2
BOUNDARY SURFACE INPUT SPECIFICATION

Flag	Surface	Boundary Surface	Required Input Data
-1	Quadric	Any listed surface	The number (s) of the surface, S(s) whose equation is to be duplicated.
0	Plane	Plane	2 boundary points (P_0 & P_1)
1	Quadric	Plane	3 non-colinear boundary points (P_i , $i = 1,2,3$)
2	Quadric	Right circular cylinder	2 axis points (P_1 & P_2) plus the radius (R)
3	Quadric	Right circular cylinder	3 boundary points (P_i , $i = 1,2,3$) plus direction numbers (l:m:n) of an element
4	Quadric	Quadric cylinder	5 boundary points (P_i , $i = 1, \dots, 5$) plus direction numbers (l:m:n) of an element
5	Quadric	Quadric	10 coefficients of the boundary surface
6	Quadric	Quadric	Bounded surface and boundary surface numbers of previously defined boundary surface.

The second case (Flag 0) is for a straight line boundary on a plane. The required input is two points on the boundary. With this information, the program computes the equation of a plane through the two points which is perpendicular to the given plane.

The third form of input (Flag 1) gives a plane boundary surface through any three given non-colinear points. This input requires the coordinates of these three points.

The next three formats are for cylindrical boundary surfaces. The input for each of these is exactly the same as for the cylindrical working surface. For an arbitrary cylinder (Flag 4), the direction numbers of an element plus five boundary points are required, while for a right circular cylinder, either two axis points and the radius (Flag 2) or three boundary points and the direction numbers of an element (Flag 3) are required.

The seventh input (Flag 5) accepts the 10 coefficients of the equation of the boundary surface directly. The last (Flag 6) permits the duplication of a previous boundary surface, from the number of the surface bounded by the previous boundary surface and the number of the boundary on this surface. Note that input Flag -1 duplicates only the surfaces listed in the surface input section, while input Flag 6 duplicates boundary equations. With these two input formats any previously defined surface or boundary surface may be duplicated.



IV - PROGRAM INPUT

This program has a maximum allowable number of surfaces, of boundaries per surface, of grid points per surface, and of interfering surfaces. These dimension limits are shown in Table 4-1.

TABLE 4-1
DIMENSION LIMITS

Quantity	Dimension
Number of quadric surfaces	100
Number of boundaries per surface	6
GS	20
GS for a plane	28
Interfering surfaces	any 10 of the quadric surfaces

The input for the program is divided into three blocks, the surface block, the boundary block, and the control block.

SURFACE BLOCK

The surface input block requires from two to six cards for each surface. Each card has room for seven fields of data, each with ten columns. All fixed point data (integers with no decimal point given) such as surface numbers and flags are input on the right side of the field, while all floating point numbers (data containing a decimal point, which may appear in any of the ten columns) such as the side index, the limits on the variables, or the

surface points are input on the left of the field. Table 4-2 shows the general input format for a surface, while Table 4-3 gives the required input for each of the 10 surface input specifications.

Card #1

The first card contains five pieces of information, all in fixed point notation. The first is the input format index, which in fixed point notation, occupies the right side of the first field, columns 1 to 10. The second bit of information is the surface number, a fixed point number, in columns 11 to 20. The flag (1 for x, 2 for y, or 3 for z) for the dependent variable occupies the right side of columns 21 to 30. Columns 31 to 40 are to be used to specify the number of boundaries which are to be given for a surface in the boundary input block. If this surface is to have the same equation as a preceding surface (i.e. if the input format flag is 8), then columns 41 to 50 will give the number of the surface whose equation is to be duplicated. Otherwise this space will contain a zero.

Card #2

The second card contains six items of information, all of which are in floating point form and are therefore at the left side of their respective fields. In order, the first is the side index from column 1 to 10. The second and third are, respectively, the minimum and the maximum values of the first independent variable assuming the variables are given in the order x, y, and z. The minimum value occupies the columns 11 to 20, while the maximum value is in columns 21 to 30. Columns 31 to 40 and 41 to 50 will give the minimum and maximum values of the second independent variable. Note that if these values occur out of order, the computation of any shape factor involving this surface may be incorrect, provided the machine will compute it. The last information on this card is the number GS in columns 51 to 60. If the engineer is interested only in computing the surface equation, the last five fields, columns 11 to 60, may be left blank. An example of this would be a surface which is to be used only as an interfering surface.



TABLE 4-2
GENERAL SURFACE INPUT FORMAT

GENERAL PURPOSE DATA SHEET												LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION												PAGE OF			
PREPARED	NAME			DATE			TITLE																		JOB NO.		
CHECKED																									GROUP		
SEQ.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	10
Card	77	80	1	5	6	10	15	20	25	30	35	40	45	46	47	48	49	50	51	52	53	54	55	56	57	58	72-73-74
1st Card	SPEC OR INDEX	SURFACE	DATA VAR.	NUMBER	OR	Y	N.B.	SURFACE																			
FOR SURFACE	NUMBER	NAME FOR	BUDYS FOR	NUMBER OF THE				STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES	STREACHES
1-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
2-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
3-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
4-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
5-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
6-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
7-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
8-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
9-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
10-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
11-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
12-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
13-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
14-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
15-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
16-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
17-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
18-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
19-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
20-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
21-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
22-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
23-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
24-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
25-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
26-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
27-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
28-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
29-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
30-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
31-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
32-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
33-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
34-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
35-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
36-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
37-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
38-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
39-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
40-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
41-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
42-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
43-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
44-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
45-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
46-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
47-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
48-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
49-ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
50-ITEM																											

TABLE 4-3
SAMPLE SURFACE INPUTS

SEQ.	GENERAL PURPOSE DATA SHEET										LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION										PAGE OF JOB NO.	
	PREPARED	NAME		DATE	TITLE		ID					NO.					EWA					
CHECKED																						GROUP
1st surface	77 80 1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	72 73 75						
1st	-1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.						
A																						
B																						
2nd surface	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
2nd	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5						
X ₁																						
Y ₁																						
Z ₁																						
Y ₂																						
Z ₂																						
Y ₃																						
Z ₃																						
3rd surface	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1						
3rd	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.						
X ₁																						
Y ₁																						
Z ₁																						
X ₂																						
Y ₂																						
Z ₂																						
Y ₃																						
Z ₃																						
4th surface	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2						
4th	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.						
X ₁																						
Y ₁																						
Z ₁																						
X ₂																						
Y ₂																						
Z ₂																						
Y ₃																						
Z ₃																						
5th surface	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3						
5th	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.						
X ₁																						
Y ₁																						
Z ₁																						
X ₂																						
Y ₂																						
Z ₂																						
Y ₃																						
Z ₃																						



TABLE 4-3 (Continued)

SEQ.	GENERAL PURPOSE DATA SHEET										LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION										PAGE OF JOB NO.			
	PREPARED	NAME	DATE	TITLE																				
CHECKED				NO EWA										GROUP										
				ID																				
17. 80.1	5	16	15	20	25	30	35	40	45	50	55	60	65	70	72.73	76								
6th surface	4	6	5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2		
1.	1.	4.	-2.	4.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.		
X ₁	X ₁	Z ₁	X ₂	Z ₂	X ₂	Z ₂	X ₂	Z ₂	X ₂	Z ₂	X ₂	Z ₂	X ₂	Z ₂	X ₂									
Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁	Y ₁			
Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁	Z ₁			
13	Z ₃	Z ₃	Z ₃	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	
	(THREE POINTS AND DIRECTIONAL NUMBERS OF A RIGHT CIRCULAR CYLINDER)																							
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	
7th surface	5	7	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	
1.	-3.	-1.	2.	5.	16.	X ₃	Z ₂	Y ₂	Z ₂	Y ₂	Z ₂	X ₂	Z ₂	Y ₂	Z ₂	X ₂	Z ₂	Y ₂	Z ₂	X ₂	Z ₂	Y ₂	Z ₂	
X ₁	Y ₁	Z ₁	X ₁	Y ₁	Z ₁	X ₁	Y ₁	Z ₁	X ₁	Y ₁	Z ₁	X ₁	Y ₁	Z ₁	X ₁	Y ₁	Z ₁	X ₁	Y ₁	Z ₁	X ₁	Y ₁		
Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃		
Z ₅	L	M	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	
3th surface	6	3.	2.	8.	3.	2.	1.	5.	4.	3.	2.	1.	0.	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.	
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.		
X ₁	Y ₁	Z ₁	X ₁	Y ₁	Z ₁	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	X ₂	Y ₂		
Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃		
Z ₅	[TWO AXIS POINTS (P ₁ AND P ₂) PLUS THREE POINTS (P ₁ = 2, 3, 4) SPACED LONGITUINALLY OF AN ARBITRARY CONIC OR REVOLUTION]																							
9th surface	7	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	
1.	Z ₄	Y ₄	X ₄	Z ₄	Y ₄	X ₄	Z ₄	Y ₄	X ₄	Z ₄	Y ₄	X ₄	Z ₄	Y ₄	X ₄	Z ₄	Y ₄	X ₄	Z ₄	Y ₄	X ₄	Z ₄		
X ₁	Y ₁	Z ₁	X ₁	Y ₁	Z ₁	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	X ₂	Y ₂	Z ₂	
Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃	X ₃	Y ₃	Z ₃		
Z ₅	[NINE SURFACE POINTS (P ₁ = 1-9) OF AN ARBITRARY CONIC]																							

TABLE 4-3 (Continued)

GENERAL PURPOSE DATA SHEET			LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION												PAGE OF JOB NO.		
PREPARED	NAME	DATE	TITLE														
CHECKED			FO												EWA		
ID			60	55	50	45	40	35	30	25	20	15	10	5	77	73	70
SEQ.																	
77	80.1																
10th																	
surface	8																
-1.			-8.	-2.	-6.	-1.											
(CONTINUING FROM THIS SURFACE AND IS EQUAL TO THE SAME PARTICIPANTS AS THOSE ON SURFACE NUMBER FOUR. THIS IS DONE IN THIS PROGRAM.)																	
LITH	5	10	15	20	25	30	35	40	45	50	55	60	65	70	77	73	70
surface	0																
(HERE THE INPUT FLAG SF > 0 WILL CALL THE PROGRAM TO TRANSFER TO THE BOUNDARY INPUT SECTION OF THE PROGRAM. SF = 11 IS A BOUNDARY SURFACE NUMBER)																	
S.	10	15	20	25	30	35	40	45	50	55	60	65	70	77	73	70	
E.	5	10	15	20	25	30	35	40	45	50	55	60	65	70	77	73	70



Card #3

The third, and if necessary, additional cards, supply the input data required for the particular input format flag used in columns 1 to 10 of the first card. These data are supplied in floating point form, seven items per card and ten columns per item (columns 1 to 10, 11 to 20, ..., 61 to 70). Inputs required to specify ten surfaces are shown in Table 4-3, and they serve as examples of each of the ten surface input formats.

Terminal Card

No card is required between surfaces, but a card giving a surface format flag ≥ 9 and a dummy surface number is required at the end of the surface input block.

BOUNDARY BLOCK

The boundary input block requires from one to four cards per boundary. These cards are also dimensioned for seven items - each requiring ten columns. Table 4-4 shows the general input format for a boundary, while Table 4-5 gives the required input for each of the eight boundary input specifications.

Card #1

The first card contains five items in fixed point notation; columns 1 to 50, plus the side index columns 51 to 60 which is in floating point notation. As in the surface input, the first item is the boundary format flag which appears in columns 1 to 10.

The second and third items, columns 11 to 20 and 21 to 30, respectively, are the surface number of the bounded surface and the number of the boundary. The fourth and fifth items, columns 31 to 40, and 41 to 50, depend on the boundary input format flag. If the input format flag is -1, then the boundary surface is to have the same equation as a surface listed in the surface input. Thus, the input in columns 31 to 40 will be the number of the surface whose equation is to be duplicated, while columns 41 to 50 will be blank. If the boundary format Flag 6 is used, the equation of this boundary surface is to duplicate the equation of the nth boundary surface of surface k. Thus, the input in columns 31 to 40 will be the number k of the surface whose nth boundary



GENERAL BOUNDARY INPUT FORMAT



TABLE 4-5
SAMPLE BOUNDARY INPUTS

SEQ.	GENERAL PURPOSE DATA SHEET												NO	EXA	PAGE OF			
	PREPARED		NAME		DATE		TITLE		LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION									
	CHECKED																	
1st Bdry	5	10	15	20	25	30	35	40	45	50	55	60	65	70	72,73,74			
(BOUNDARY B(1,1)) IS SET EQUAL TO THE SPECIFIC ITEMS OF SURFACE NUMBER 2)																		
2nd Bdry	0	2	1	2	1	2	1	2	1	0	1.0							
X ₁	Y ₁	Z ₁	X ₂	Y ₂	Z ₂	X ₃	Y ₃	Z ₃	X ₄	Y ₄	Z ₄	X ₅	Y ₅	Z ₅				
1	5	10	15	20	25	30	35	40	45	50	55	60	65	70				
TWO BOUNDARY POINTS (P ₁ & P ₂) ON PLANE, FROM WHICH BOUNDARY COORDINATES FOR B(2,1) ARE CALCULATED																		
3rd Bdry	1	3	2	1	2	1	3	2	1	0	-1.0							
X ₁	Y ₁	Z ₁	X ₂	Y ₂	Z ₂	X ₃	Y ₃	Z ₃	X ₄	Y ₄	Z ₄	X ₅	Y ₅	Z ₅				
1	5	10	15	20	25	30	35	40	45	50	55	60	65	70				
4TH NON-COPLANAR BOUNDARY POINTS (P ₁ = 1,2,3) FROM WHICH BOUNDARY CONNECTIONS FOR B(3,2) ARE CALCULATED																		
X ₃	Y ₃	Z ₃	X ₁	Y ₁	Z ₁	X ₂	Y ₂	Z ₂	X ₃	Y ₃	Z ₃	X ₄	Y ₄	Z ₄				
1	5	10	15	20	25	30	35	40	45	50	55	60	65	70				
5th Bdry	2	3	3	2	1	1	2	1	0	-1.0								
X ₁	Y ₁	Z ₁	X ₂	Y ₂	Z ₂	X ₃	Y ₃	Z ₃	X ₄	Y ₄	Z ₄	X ₅	Y ₅	Z ₅				
1	5	10	15	20	25	30	35	40	45	50	55	60	65	70				
TWO AXIS POINTS (P ₁ & P ₂) AND RADIUS R OF RIGHT CIRCULAR CYLINDER BOUNDARY COEFFICIENTS FOR B(3,3) ARE CALCULATED																		
5th Bdry	3	4	4	3	2	1	1	2	1	0	1.0							
X ₁	Y ₁	Z ₁	X ₂	Y ₂	Z ₂	X ₃	Y ₃	Z ₃	X ₄	Y ₄	Z ₄	X ₅	Y ₅	Z ₅				
1	5	10	15	20	25	30	35	40	45	50	55	60	65	70				
THREE BOUNDARY POINTS (P ₁ = 1,2,3) AND DIRECTIONAL NOS. 1, 2, 3, OR RIGHT CIRCULAR CYLINDER BOUNDARY COEFFICIENTS FOR X(3,1) ARE CALCULATED																		



TABLE 4-5 (Continued)



surface is to be duplicated and columns 41 to 50 will contain the number n of the boundary surface of surface number k. Columns 51 to 60 will contain the side index of the boundary in floating point notation.

Card #2, etc.

The additional cards required for boundary input format Flags 0, ..., 5 supply the data required by each of these formats - seven items per card in fields of ten columns. The data are given in floating point notation and must be supplied in the order indicated by the format. Examples of each of the eight input formats are given in Table 4-5.

Terminal Card

While no cards are required to separate the data for individual boundaries, a card giving an input flag ≥ 7 , a dummy surface number, and a dummy boundary number is required at the end of the boundary block.

CONTROL BLOCK

The last block of input controls the computation in the program. While the surface and boundary surface equations are computed automatically, all other computations must be requested. The input format is shown in Table 4-6. The computation and storage of the grid points on a surface, the surface to surface shape factors, the point to surface shape factors and the computation of surface areas are all included. Samples of each of these are shown in Table 4-7. Note that the grid points for a surface must be computed before a surface area can be used in any of the other three types of computations. The only exceptions are those surfaces which are to be used only as interfering surfaces or surfaces containing the point in a point to surface shape factor.

Grid Point Computation

Control Flags -1 and 0 call for the computation and storage on tape of grid points. Flag -1 is used if there are no previously computed grid points on tape. Once the machine has computed and stored grid points on tape it must search for the end of this storage block before storing additional grid points



TABLE 4-6
COMPUTATION INPUT



TABLE 4-7
SAMPLE COMPUTATION INPUTS



TABLE 4-7 (Continued)



of additional surfaces. Thus Flag -1 is used only on the first card in the control section and Flag 0 is used to call for the grid points of all additional surfaces. All the data on the grid point computation cards are in fixed point notation, and hence appear on the right side of the indicated field.

Card #1 - Since there are no previously stored grid points, control Flag -1 appears in the columns 9 and 10 on this card to indicate that the grid points of the surfaces $S(s)$ and $S(t)$. The surface number s and t occur as fixed point numbers in the columns 11 to 20 and columns 21 to 30 respectively.

Card #2, etc. - Since grid points have been computed and stored according to the instructions of the first card in the control block, control Flag 0 appears in column 10. This control flag tells the machine to compute the grid points for surfaces $S(s)$ and $S(t)$, to search for the last set of stored grid points and to add the grid points of surfaces $S(s)$ and $S(t)$ to those already stored. The surface numbers s and t appear as fixed point numbers in columns 11 to 20 and 21 to 30 respectively.

Surface-to-Surface Shape Factors

The computation of the shape factor $F(s,t)$ from the surface $S(s)$ to surface $S(t)$ for any surfaces $S(s)$ and $S(t)$ whose grid points have been computed requires control Flag 1. The machine then computes and prints out the product $A(s) \cdot F(s,t)$ of the area of surface $S(s)$ times the shape factor $F(s,t)$, the area of surface $S(s)$ and the shape factor $F(s,t)$. If the n interfering surfaces $S(s_1)$ to $S(s_n)$ are listed, the machine will compute the shape factor with interference and add "With interference by $S(s_1)$ $S(s_n)$," to the printout.

At least one card is required for each shape factor. If the number n of interfering surfaces is not zero, additional cards will be required.

Card #1 - This card contains the control Flag 1 in column 10 with the surface numbers s and t in fixed point notation in columns 11 to 20 and 21 to 30, respectively. This calls for the shape factor $F(s,t)$ plus $A(s)F(s,t)$ and $A(s)$. The number of interfering surfaces appears in columns 31 to 40 as an integer.



Card #2, etc. - If the number n of interfering surfaces is not zero, the surface numbers s_1 to s_n are given in one or two additional cards as are required. These surface numbers are given in fixed point notation - seven numbers on the first card in columns 1 to 10, columns 11 to 20, etc., and 3 numbers on the second card. The maximum number of interfering surfaces is 10.

Point-to-Surface Shape Factor

Control Flag 2 is used for the shape factor from a point $P(i)$ on surface $S(s)$ to any surface $S(t)$ whose grid points are in storage. The machine will compute the dependent variable of point $P(i)$ and the shape factor from a differential area located on surface $S(s)$ at point $P(i)$ to surface $S(t)$. The printout gives the coordinates of $P(i)$ and the shape factor $P(i)F(s,t)$. If the n interfering surfaces $S(s_1)$ through $S(s_n)$ are specified, the program will account for the interference and add the interference printout. The control cards for the point-to-surface shape factors require at least three cards.

Card #1 - Control Flag 2 appears in column 10 and the fixed point surface numbers s and t in columns 11 to 20, and 21 to 30, respectively. Columns 31 to 40 contain the number of interfering surfaces. This calls for the computation of one or more point-to-surface shape factors from points on $S(s)$ to $S(t)$.

Card #2 - The first entry, in column 10, is a flag (1 for x , 2 for y , or 3 for z) indicating the dependent variable to be used on surface $S(s)$. The number j of points $P(i_1)$ to $P(i_j)$ on surface $S(s)$ for which point-to-surface shape factors are to be computed to surface $S(t)$ appears in fixed point notation in columns 11 to 20. Note that spheres and similar surfaces may yield up to $2j$ points, as illustrated in Example Problem 3. In columns 21 to 30, the side index for surface $S(s)$ is given in floating point notation. Note that the dependent variable indicated in column 10 and the side index in columns 21 to 30 is not required to agree with those used in any other listing of surface $S(s)$, so they should be chosen to match the points $P(i_1)$ to $P(i_j)$. If any of these points are vertical points on the surface relative to the chosen dependent variable, additional point-to-surface shape factors inputs should be made, each with the appropriate choice of dependent variable. For example, on

the sphere $x^2 + y^2 + z^2 - 4 = 0$, used as the surface $S(5)$ in the third example problem, the three points $P(1) = (2,0,0)$, $P(2) = (0,2,0)$ and $P(3) = (0,0,2)$ all require a different choice of dependent variable. Thus $P(1)F(5,1)$, $P(2)F(5,1)$, and $P(3)F(5,1)$ must be input separately rather than being input in a single request.

Card #3, etc. - The independent coordinates of the points $P(i_1)$ to $P(i_j)$ are given seven coordinates per card in fields of 10 (columns 1 to 10, 11 to 20, etc.) using floating point notation.

Interfering Surface Cards

If the number n of interfering surfaces is not zero, additional cards must be supplied giving the surface numbers s_1 to s_n of the interfering surfaces $S(s_1)$ to $S(s_n)$. These numbers are given seven to the card in fields of 10 columns.

Surface Area

The computation of surface area is specified one surface per card. The control Flag 3 in column 10 and the fixed point number s in columns 11 to 20 require the computation and printout of the area of surface $S(s)$.

End of Input

A card with control Flag 4 in column 10 ends the input.



V - PROGRAM OUTPUT

The output from this program is fully identified as to which surfaces or points "view" and which are "viewed", the viewing surface or point designated by s and the viewed surface by t. Both area-shape factor products and areas alone are printed.

Tables 5-1 through 5-4 contain typical output information. The first information appearing for each surface or boundary (Tables 5-1 and 5-2) is a duplication of the input for that surface or boundary. This is followed by the surface reference point, $Q = (x,y,z)$ for all surfaces which are not duplicates of a previous surface. If the surface or boundary equation is determined by the computer, the residuals obtained by substituting the coordinates of the surface points (which were used to define the surface equation) into the equation are printed below the reference point Q. In Tables 5-1 and 5-2 all equations were input, so no such numbers exist. If one or more of these values approaches the magnitude of the coefficients, which are printed out for each surface, the limited accuracy of the digital solution has probably resulted in an invalid equation. The magnitude of these residuals should be compared with the coefficients to make sure all invalid equations have been rejected.

Following these surface and boundary summary sheets is a list of computed shape factors. This format depends on whether surface-to-surface or point-to-surface shape factors are computed.

SURFACE-TO-SURFACE SHAPE FACTORS

A typical surface-to-surface shape factor output sheet is shown in Table 5-3. The line beginning with "DELS" includes the increments computed for the independent variables projected rectangle. The second line consists of the surface numbers s and t and their area-view factor product, $A(s)F(s,t)$. The third line of output gives the surface s and its area $A(s)$. The fourth

line repeats the surfaces considered and follows with the shape factor, $F(s,t)$.

The last line in each distinct group is actually the computational input for the next set of outputs. If interference by surfaces s_1 through s_j is specified for the shape factor $F(s,t)$, one or more lines are added above the "DELS" line. These give the surface numbers s and t plus the printout " $F(s,t)$ with interference by surface numbers s_1, \dots, s_j "

POINT-TO-SURFACE SHAPE FACTORS

Table 5-4 shows the output for several point-to-surface shape factors. As for the surface-to-surface output, the results are preceded by the computational input. Immediately preceding the first line of output is a block of independent variables for the points on the viewing surface (in this example, surface 5).

The output results begin with the point number i , followed by the viewing surface, s , the viewed surface, t , and the shape factor $P(i)F(s,t)$. The second line contains the coordinates of $P(i)$. If interfering surfaces are given, information identical to that given in the surface-to-surface output is added above the first line.



TABLE 5-1. SURFACE SPECIFICATION

-1.000	-18.031	1.	18.031	1.	38.000	2	76.000	C	16.000
0.	1.000	0.	0.	0.	0.	0.	0.	C.	C.
Q=(0,0,0) FOR SURFACE NO. 1									
C= -0.	G= -0.	D= 0.	H= -0.	A= -0.	E= -0.15378700E-02	I= -C.	H= -0.15378700E-02	F= -0.	J= 0.09999999E 01
8	-18.031	18.031	2	38.000	1	76.000	1	J= 0.09999999E 01	
-1.000	-18.031	18.031	2	A= -0.15378700E-02	B= -0.15378700E-02	C= -C.	D= -0.15378700E-02	E= -C.	F= -C.
0.	-0.	0.	0.	E= -C.	I= -C.	J= 0.09999999E 01	F= -C.	G= -C.	H= -C.
-1.000	-18.031	18.031	1	38.000	1	76.000	1	J= 0.09999999E 01	
C= -0.	G= -0.	D= 0.	H= -0.	A= -0.15378700E-02	B= -0.15378700E-02	C= -C.	D= -0.15378700E-02	E= -C.	F= -C.
8	-18.031	18.031	0.	38.000	1	76.000	1	J= 0.09999999E 01	
-1.000	-18.031	18.031	0.	A= -0.15378700E-02	B= -0.15378700E-02	C= -C.	D= -0.15378700E-02	E= -C.	F= -C.
0.	-0.	0.	0.	E= -C.	I= -C.	J= 0.09999999E 01	F= -C.	G= -C.	H= -C.
1.000	0.	0.	0.	1	38.000	1	76.000	1	J= 0.09999999E 01
Q=(0,0,0) FOR SURFACE NO. 5									
C= 0.	G= 0.	D= 0.	H= 0.	A= 0.	E= 0.	I= C.	C= 0.	F= 0.	J= 0.09999999E 01
-1	-0.	5	2	0.	0.	C.	C.	C.	C.
1.000	-0.	-0.	-0.	-0.	-0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1.000	0.	29.000							
-1.CCO	0.	0.	0.	0.	0.	0.	0.	0.	0.
Q=(0,0,0) FOR SURFACE NO. 6									
C= 0.	G= 0.	D= 0.	H= 0.	A= 0.	E= 0.	I= C.	C= 0.	F= 0.	J= 0.09999999E 01
-1	-0.	6	1	0.	0.	C.	C.	C.	C.
1.000	-0.	-0.	-0.	-0.	-0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-1.CCO	0.	0.	0.	0.	0.	0.	0.	0.	0.
Q=(0,0,0) FOR SURFACE NO. 9									
C= 0.	G= 0.35679681E-01	D= 0.	H= -0.12224939E-01	A= 0.	E= 0.	I= -C.	C= 0.	F= 0.	J= 0.09999999E 01
-1	-0.	9	-0	-0	-0	-0	-0	-0	-0



TABLE 5-2. BOUNDARY SPECIFICATION

5	1	1	0	C	1.C00
1.000	-1.C00	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
Q=(1,0,0) FCR BCRY NO. 1 OF SURFACE NO. 1					
COEFFICIENTS FOR HORY NC. 1 OF SURFACE NO. 1					
C= 0.	D= 0.	E= 0.	A= C.	B= 0.	-0.09999999E 01
G= 0.	H= 0.	I= 0.	F= 0.	J= 0.	
5 1 2 0. 0. 0. -1.000					
0. 0. 0. 0. 0. 0. 0. 0. 0.					
Q=(C,0,0) FCR BCRY NC. 2 OF SURFACE NO. 1					
COEFFICIENTS FOR HORY NC. 2 OF SURFACE NO. 1					
C= 0.34226038E-03 D= 0.	E= 0.	A= 0.	B= 0.		
G= 0.	H= 0.	I= 0.	F= 0.		
6 2 1 0. 0. 1. 0. 0. 0.					
Q=0 COEFFICIENTS FOR HORY NC. 1 OF SURFACE NO. 2					
C= 0.	D= 0.	E= 0.	A= C.	B= 0.	-0.09999999E 01
G= 0.	H= 0.	I= 0.	F= 0.	J= 0.	
6 2 2 0. 0. 2. 0. 0. 0.					
Q=0 COEFFICIENTS FOR HORY NC. 2 OF SURFACE NO. 2					
C= 0.34226038E-03 D= 0.	E= 0.	A= 0.	B= 0.		
G= 0.	H= 0.	I= 0.	F= 0.		
6 3 1 0. 0. 1. 0. 0. 0.					
Q=(0,0,1) FOR BCRY NO. 2 OF SURFACE NO. 3					
COEFFICIENTS FOR HORY NC. 2 OF SURFACE NO. 3					
C= 0.	D= 0.	E= 0.	A= C.	B= 0.	-0.09999999E 01
G= 0.	H= 0.	I= 0.	F= 0.	J= 0.	
5 3 2 0. 0. 0. 0. 0. 0.					
0. 0. 0. 0. 0. 0. 0. 0. 0.					
Q=(0,0,1) FOR BCRY NO. 1 OF SURFACE NO. 3					
COEFFICIENTS FOR HORY NC. 1 OF SURFACE NO. 3					
C= -0.27027027E-01 D= 0.	E= 0.	A= C.	B= 0.	-0.09999999E 01	
G= -0.	H= 0.	I= 0.	F= 0.	J= 0.	
6 4 1 0. 0. 1. 0. 0. 0.					
0. 0. 0. 0. 0. 0. 0. 0. 0.					
Q=-0.27027027E-01 FOR HORY NC. 2 OF SURFACE NO. 4					
COEFFICIENTS FOR HORY NC. 2 OF SURFACE NO. 4					
C= -0.27027027E-01 D= 0.	E= 0.	A= -0.	B= -0.		
G= -0.	H= 0.	I= 0.	F= 0.	J= 0.	
7 9 1 0. 0. 0. 0. 0. 0.					
0. 0. 0. 0. 0. 0. 0. 0. 0.					



TABLE 5-3. SURFACE-TO-SURFACE SHAPE FACTORS

DELS	0.13256250E-00	0.11250000E-00	0.37500000E-00	0.37500000E-00
S= 20	T= 28	A(S)F(S,T)=	0.2350722E 01	
S= 20	AREA (S)=	0.40328450E 01		
S= 20	T= 28	F(S,T)=	0.53446885E 00 -0	-0
1	21			
DELS	0.98750000E-01	0.18250000E-00	0.37500000E-00	0.37500000E-00
S= 21	T= 28	A(S)F(S,T)=	0.11544086E 01	
S= 21	AREA (S)=	0.40014727E 01		
S= 21	T= 28	F(S,T)=	0.26849594E-00 -0	-0
1	21			
DELS	0.13256250E-00	0.35000000E-01	0.62500000E 00	0.62500000E 00
S= 1	T= 29	A(S)F(S,T)=	0.10470989E 01	
S= 1	AREA (S)=	0.13771351E 01		
S= 1	T= 29	F(S,T)=	0.76934576E 00 -0	-0
1	21			
DELS	0.12187500E-00	0.36249999E-01	0.62500000E 00	0.62500000E 00
S= 2	T= 29	A(S)F(S,T)=	0.10808427E 01	
S= 2	AREA (S)=	0.13125870E 01		
S= 2	T= 29	F(S,T)=	0.82347512E 00 -0	-0
1	3			
DELS	0.10625000E-00	0.41249999E-01	0.62500000E 00	0.62500000E 00
S= 3	T= 29	A(S)F(S,T)=	0.11076140E 01	
S= 3	AREA (S)=	0.13376735E 01		
S= 3	T= 29	F(S,T)=	0.92801517E 00 -0	-0
1	4			
DELS	0.98750000E-01	0.47499997E-01	0.62500000E 00	0.62500000E 00
S= 4	T= 29	A(S)F(S,T)=	0.98794685E 00	



TABLE 5-3. POINT-TO-SURFACE SHAPE FACTORS

	-1	1	2	-0	-c	
SURFACE NUMBER	1	WAS PUT ON TAPE				
SURFACE NUMBER	2	WAS PUT ON TAPE				
0	3	4	-0		-c	
SURFACE NUMBER	3	WAS PUT ON TAPE				
SURFACE NUMBER	4	WAS PUT ON TAPE				
2	5	1	0		-c	
2	21	1.000				
27.750	69.000	-7.750	47.000	27.750	36.000	27.750
0.	27.750	-30.000	27.750	-47.000	27.750	-68.000
9.500	63.000	9.500	49.000	9.500	36.000	9.500
0.	9.500	-30.000	9.500	-49.000	9.500	-68.000
-8.825	65.000	-6.825	49.000	-8.825	36.000	-8.825
0.	-8.825	-30.000	-8.825	-49.000	-8.825	-68.000
I= 1 S= 5 T= 1 P(I)F(S,T)= 0.22987637E-03 POINT ON S IS X(I)= 0.27750000E 02 Y(I)= -0.27979999E 02 Z(I)= 0.68000000E 02						
I= 2 S= 5 T= 1 P(I)F(S,T)= 0.24737491E-03 POINT ON S IS X(I)= 0.27750000E 02 Y(I)= -0.27999999E 02 Z(I)= 0.45999999E 02						
I= 3 S= 5 T= 1 P(I)F(S,T)= 0.69030771E-01 POINT ON S IS X(I)= 0.27750000E 02 Y(I)= -0.27999999E 02 Z(I)= 0.30000000E 02						
I= 4 S= 5 T= 1 P(I)F(S,T)= 0.42129555E-02 POINT ON S IS X(I)= 0.27750000E 02 Y(I)= -0.27999999E 02 Z(I)= 0.						
I= 5 S= 5 T= 1 P(I)F(S,T)= 0.74932462E-03 POINT ON S IS X(I)= 0.27750000E 02 Y(I)= -0.27999999E 02 Z(I)= -0.30000000E 02						
I= 6 S= 5 T= 1 P(I)F(S,T)= 0.33556215E-03 POINT ON S IS X(I)= 0.27750000E 02 Y(I)= -0.27999999E 02 Z(I)= -0.48999999E 02						
I= 7 S= 5 T= 1 P(I)F(S,T)= 0.1720357E-03 POINT ON S IS X(I)= 0.27750000E 02 Y(I)= -0.27999999E 02 Z(I)= -0.68000000E 02						
I= 8 S= 5 T= 1 P(I)F(S,T)= 0.71419627E-03 POINT ON S IS X(I)= 0.95000000E 01 Y(I)= -0.27999999E 02 Z(I)= 0.68000000E 02						
I= 9 S= 5 T= 1 P(I)F(S,T)= 0.7e860525E-03 POINT ON S IS X(I)= 0.95000000E 01 Y(I)= -0.27999999E 02 Z(I)= 0.48999999E 02						
I= 10 S= 5 T= 1 P(I)F(S,T)= 0.12455949E-03 POINT ON S IS X(I)= 0.95000000E 01 Y(I)= -0.27999999E 02 Z(I)= 0.30000000E 02						
I= 11 S= 5 T= 1 P(I)F(S,T)= 0.91592011E-05 POINT ON S IS X(I)= 0.95000000E 01 Y(I)= -0.27999999E 02 Z(I)= 0.						
I= 12 S= 5 T= 1 P(I)F(S,T)= 0.13517390E-05 POINT ON S IS X(I)= 0.95000000E 01 Y(I)= -0.27999999E 02 Z(I)= -0.30000000E 02						
I= 13 S= 5 T= 1 P(I)F(S,T)= 0.59542524E-06 POINT ON S IS X(I)= 0.95000000E 01 Y(I)= -0.27999999E 02 Z(I)= -0.48999999E 02						
I= 14 S= 5 T= 1 P(I)F(S,T)= 0.30244735E-06						



VI - EXAMPLE PROBLEMS

Three example problems are presented in this section. A detailed discussion of the problems and the reason for including each follows.

The first problem is an exercise showing each of the inputs discussed in Section IV with examples of a variety of surfaces. The second example, which approximates a rocket nozzle and storage tank problem, illustrates the effect of the choice of the dependent variable on computed surface area with the resultant effect on the shape factors. The third problem consists of a sphere inside a cube. This problem has the advantage that all areas and shape factors may be computed exactly as a check on the accuracy of the program. In computing the shape factors from one side of the cube to another, the effect of interference by the sphere is illustrated. In addition, this problem shows the effect of the choice of dependent variable on a shape factor where the area is not affected.

The values of grid size (GS) for these examples were based on the former limits of 16 or 20, depending on the type of surface (see Section III). The capacity is now 20 or 28. The old maximum values of GS were used in Examples 1 and 2. Somewhat lower values were used in Example 3 to demonstrate the accuracy obtainable with modest GS values. The discussion of Example 3 includes a comparison of grid size, accuracy, and computer time.

EXAMPLE PROBLEM #1

The first example problem contains ten surfaces and eight boundary surfaces showing examples of each input format plus examples of each type of computation.

Surface Input

Surface #1 - Arbitrary Conic - Surface #1 is the lower side of a plane circular disk of unit radius lying on the xy-plane with its center at the



origin as shown in Figure 6-1. Since this surface is a section of a plane, it can be seen from Table 4-2 (General Surface Input Formats) that surface #1 must be defined by equation or by three points. In this case, the ten coefficients A to J of the equation $z = 0$ (the xy-plane) and format Flag -1 is used. A dependent variable of z is required on this plane so the dependent variable flag is 3. The boundary surface $x^2 + y^2 + z^2 - 1 = 0$ is used to cut the plane down to the given disk so one boundary is required. The given plane $z = 0$ contains the points $(0,0,0)$, $(1,0,0)$, and $(0,1,0)$; therefore the surface reference point is $(0,0,1)$. Since the lower side (which faces away from $(0,0,1)$) is required, the side index is -1.0. The surface area will be used in a shape factor computation, therefore the limits on the independent variables and the grid size number GS are required. The minimum value for x on this disk is -1 while the maximum value of x is 1. For y the minimum and maximum values are also -1 and 1. The number GS is set at 16. The input for this surface is:

SF	S	D.V.	NB				
-1	1	3	1				
SS1	MIN-1	MAX-1	MIN-2	MAX-2		GS	
-1.	-1.	1.	-1.	+1.		16.	
A	B	C	D	E	F	G	
0.	0.	0.	0.	0.	0.	0.	
H	I	J					
0.	1.	0.					

Surface #2 - Plane - The second surface is the top of a section of the plane $6x + 4y - 3z - 12 = 0$ given in Figure 6-2. The required area has one linear and one elliptic boundary. Input Flag 0 and the three surface points (not all on a straight line) $(2,0,0)$, $(0,3,0)$, and $(0,0,-4)$ are used to define surface 2. The dependent variable z is chosen on this surface, requiring Flag 3 for the dependent variable. Note that any variable could be chosen for this surface. Two boundaries are required. The reference point is $(0,0,0)$, and since the plane lies below this reference point, side index 1.0 is used. Since grid points are required for this surface, limits on the independent



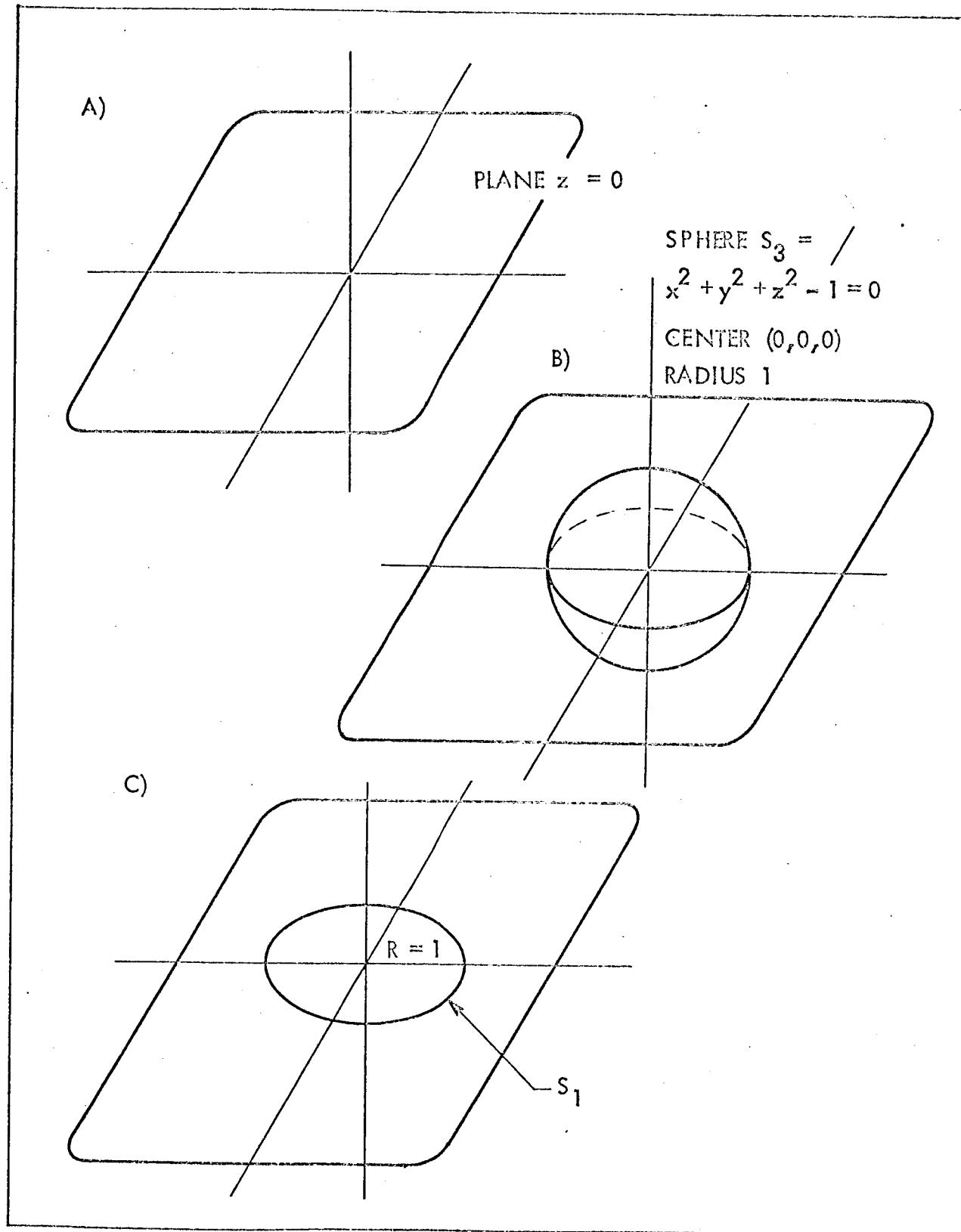


Figure 6-1. Example Problem #1 - Surface #1

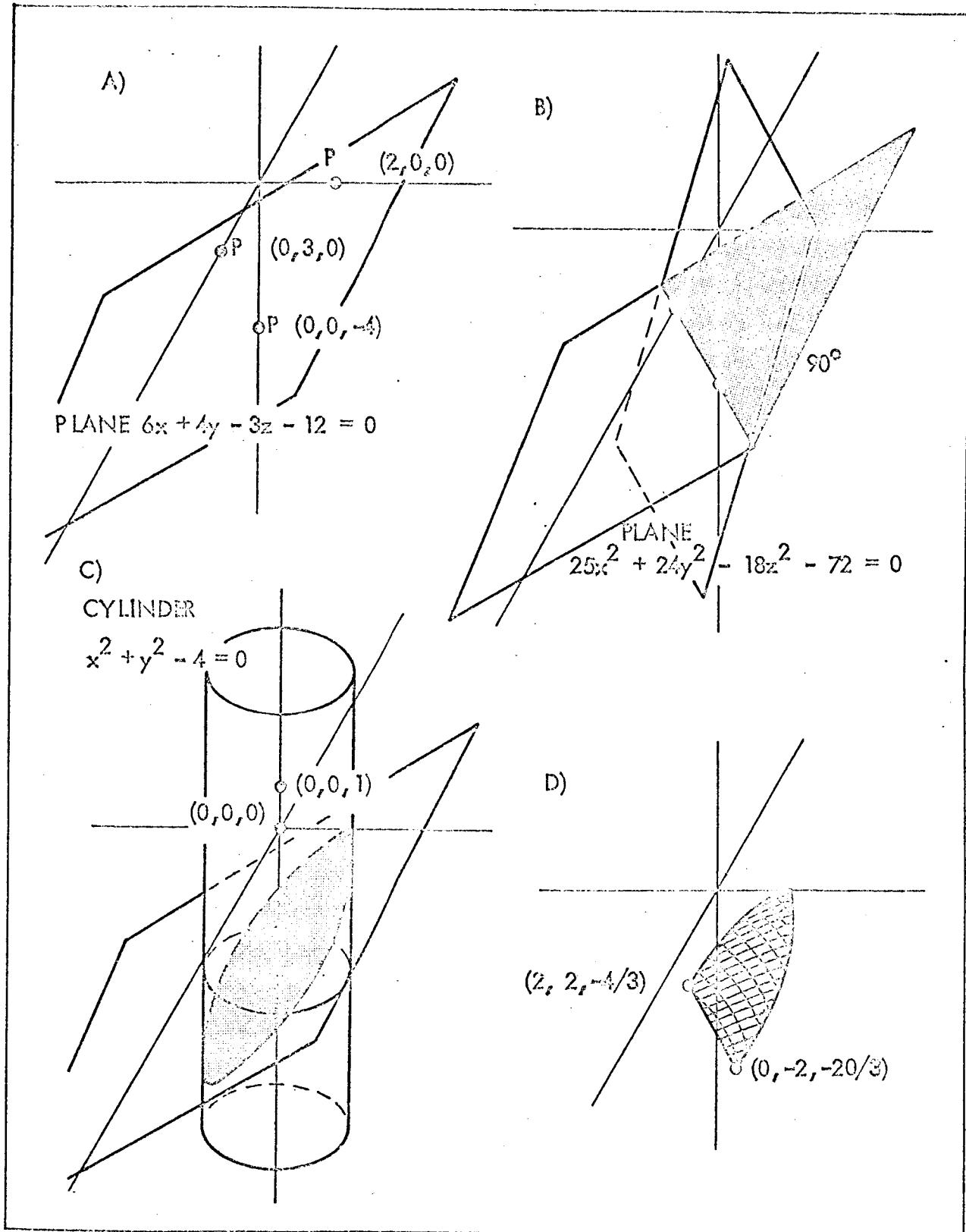


Figure 6-2. Example Problem #1 - Surface #2

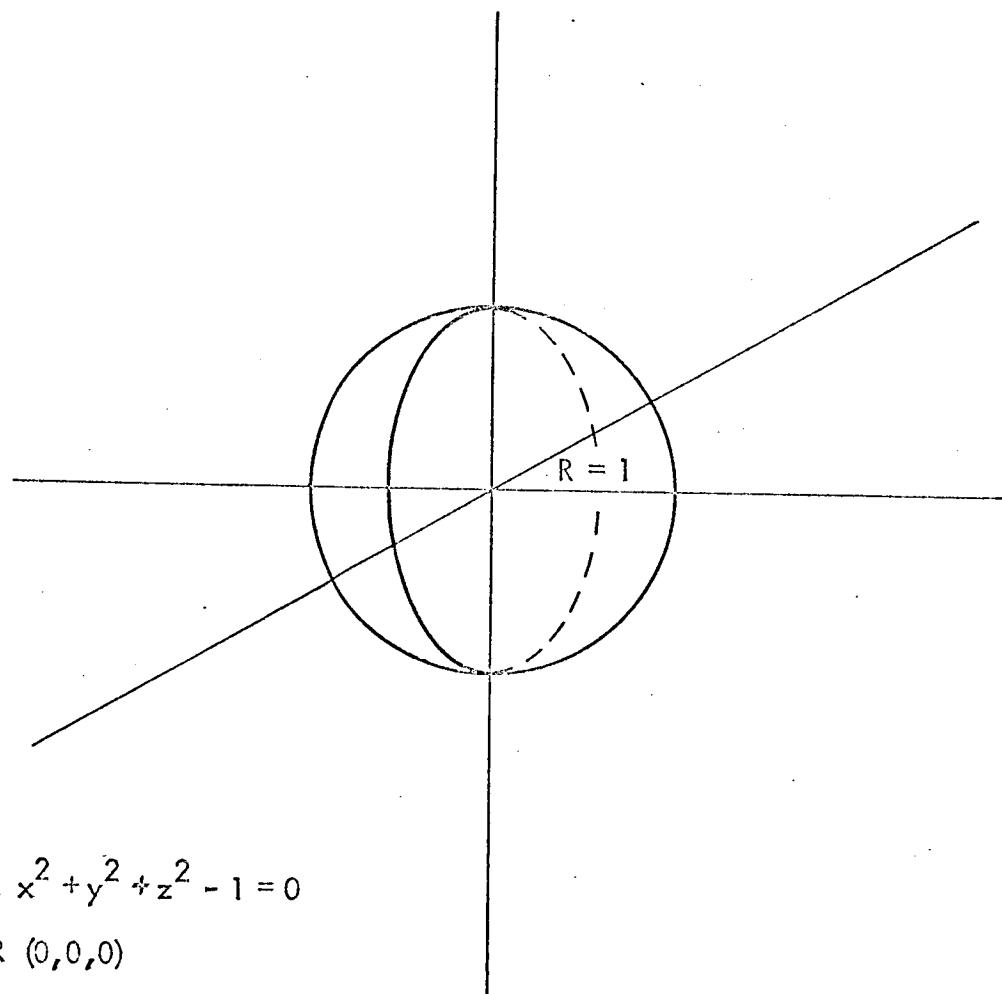
variables $0 \leq x \leq 2$ and $-2 \leq y \leq 2$ are given. The number GS is set equal to 16. Thus the input for this surface is:

SF	S	DV	NB
0	2	3	2
SSI	MIN-1	MAX-1	MIN-2
1.	0.	2.	-2.
x_1	y_1	z_1	x_2
2.	0.	0.	0.
y_3	z_3		y_2
0.	-4.		z_2
			x_3
			0.

Surface #3 - Sphere - Surface #3 is the inside of the sphere $x^2 + y^2 + z^2 - 1 = 0$, illustrated in Figure 6-3. With input Flag 1, the sphere is defined by the center (0,0,0) and the radius 1.0. On this sphere, the choice of independent variable is arbitrary and x (Flag 1) is used. No boundaries are required as the entire sphere is required. The reference point is (0,0,0), therefore the side index is 1.0 for the inside. This surface is not used in any shape factor or area computation, so the remainder of the second card is left blank. Thus input for this case is:

SF	S	DV	NB
1	3	1	0
SSI			
1.0			
x	y	z	R
0.	0.	0.	1.

Surface #4 - Sphere - The fourth surface consists of the inner (concave) side of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z - 35 = 0$, bounded by a plane and a right circular cylinder as given in Figure 6-4. Input Flag 2 plus the four surface points (7,0,0), (7,-5,5), (7,1,5), and (-2,4,5) (not all on a plane) define the surface. For this surface, the dependent variable is x (Flag 1) and two boundaries are used. The reference point is the origin,



$$\text{SPHERE } x^2 + y^2 + z^2 - 1 = 0$$

CENTER $(0, 0, 0)$

RADIUS = 1

Figure 6-3. Example Problem #1 - Surface #3

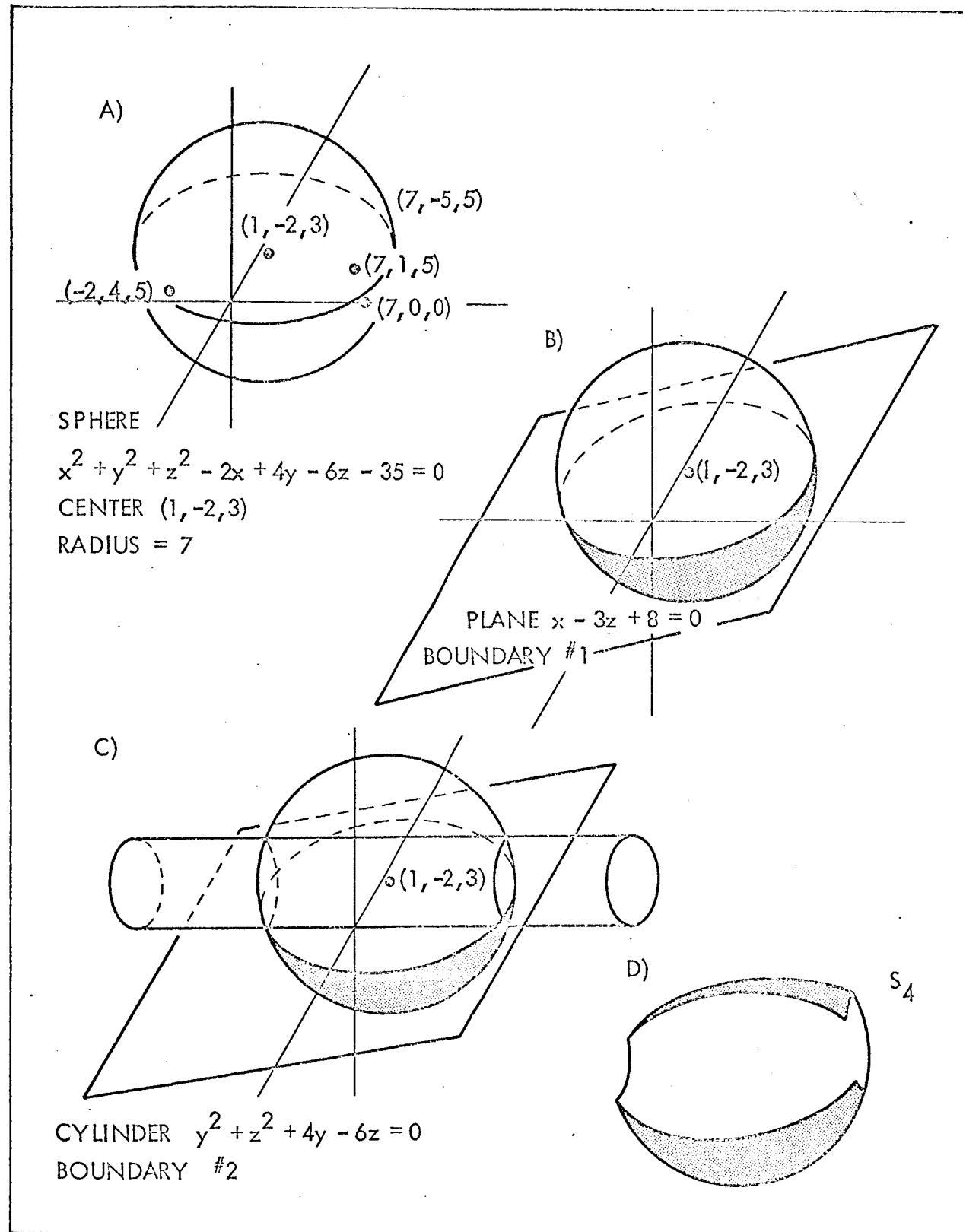


Figure 6-4. Example Problem #1 - Surface #4

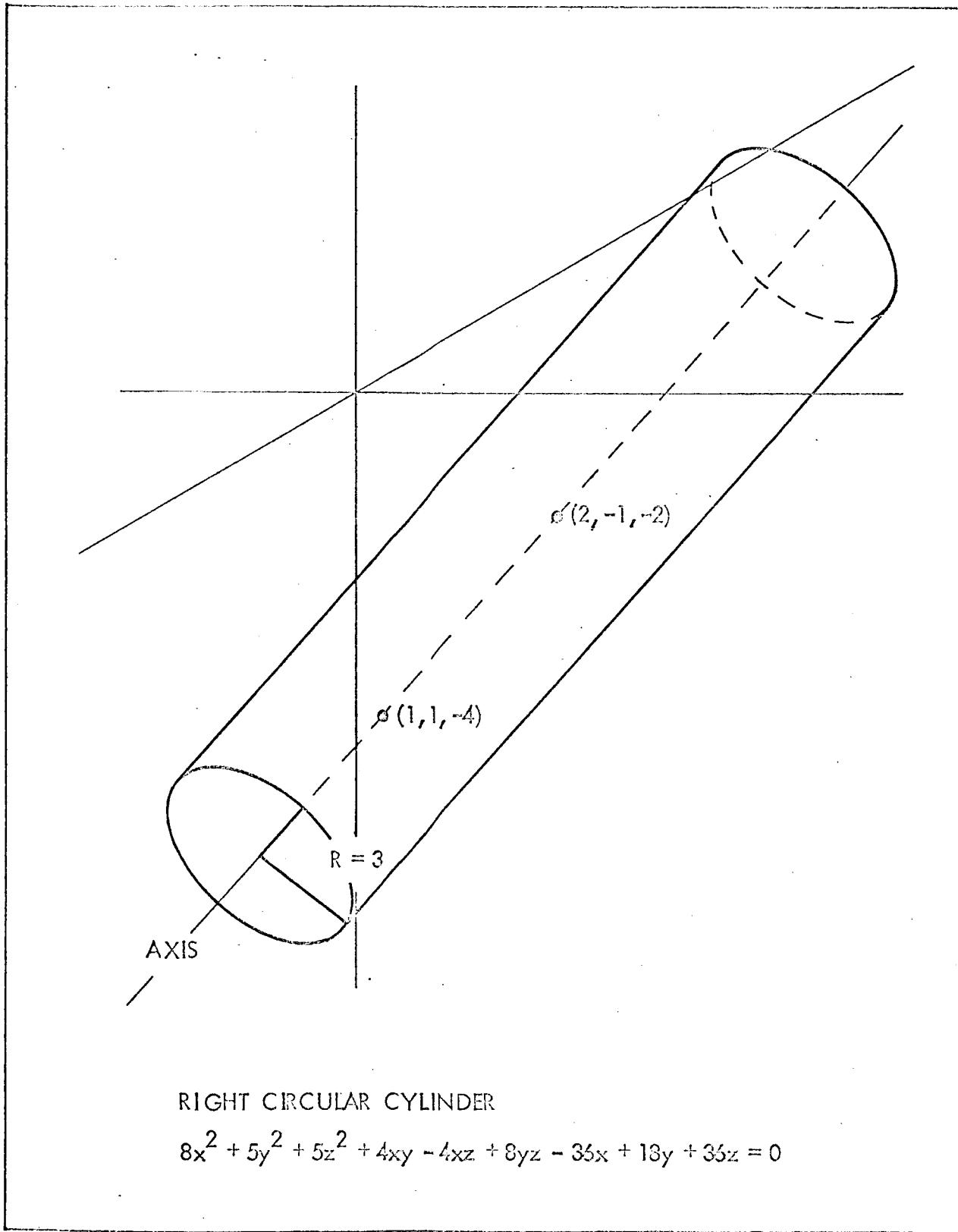
which is inside the sphere, and therefore side index 1.0 is used. The remainder of the second card for this surface is left blank since the surface is not used in the shape factor computations. The input is:

SF	S	DV	NB
2	4	1	2
SSI			
1.			
x_1	y_1	z_1	x_2
7.	0.	0.	7.
y_3	z_3	x_4	y_4
1.	5.	-2.	4.
			5.

Surface #5 - Right Circular Cylinder - Surface #5 is the unbounded right circular $8x^2 + 5y^2 + 5z^2 - 4xz + 4xy + 8yz - 36x + 18y + 36z = 0$ in Figure 6-5. Note that an unbounded surface such as this, or either of the next two surfaces, cannot be used in the shape factor computations except as an interfering surface or as the base surface (the surface containing the point) for a point-to-surface shape factor. This surface is defined by input Flag 3, the axis points $(1,1,-4)$ and $(2,-1,-2)$, plus the radius $R = 3$. The dependent variable is x , the surface has no boundaries, and a side index 1.0 is given. Note that with the reference point $(1,0,0)$ indicated by the equation, this defines the inside of the cylinder. But with the small constant computed by the program, the reference point is changed to $(0,0,0)$, and with the positive side index, the outside is actually given. The input for this surface is:

SF	S	DV	NB
3	5	1	0
SSI			
1.			
x_1	y_1	z_1	x_2
1..	1.	-4.	2.
			-1.
			-2.
			3.





RIGHT CIRCULAR CYLINDER

$$8x^2 + 5y^2 + 5z^2 + 4xy - 4xz + 8yz - 35x + 13y + 35z = 0$$

Figure 6-5. Example Problem #1 - Surface #5

Surface #6 - Right Circular Cylinder - The unbounded cylinder $x^2 + z^2 - 2x + 6z - 15 = 0$ is shown in Figure 6-6 as surface #6. This cylinder is defined by input Flag 4, the three surface points (no two on the same element) (1,1,2), (-3,0,0), and (4,2,1), plus the direction numbers of an element (0 : 1 : 0). On this cylinder, the choice of dependent variable is restricted to x and z (x is given) since the elements of the cylinder are parallel to the y-axis. With the reference point (0,0,0) inside the cylinder, the side index 1.0 specifies the inside. The input is:

SF	S	DV	NB			
4	6	1	0			
SSI						
1.0						
x_1	y_1	z_1	x_2	y_2	z_2	x_3
1.	1.	2.	-3.	0.	0.	4.
y_3	z_3	L	M	N		
2.	1.	0.	1.	0.		

Surface #7 - Quadric Cylinder - The hyperbolic cylinder $y^2 - z^2 + 64 = 0$, surface #7, is illustrated in Figure 6-7. With input Flag 5, this surface is defined by the five surface points (no two on the same element) (0,0,8), (0,6,10), (0,-6,10), (0,15,17), and (0,-15,17) plus the direction numbers (1 : 0 : 0) of an element. Since the elements are parallel to the x-axis and the tangent planes along the line $y = 0$, $z = 8$ are horizontal, z is the only choice of dependent variable. Relative to the reference point (0,0,0), the side index -1.0 indicates the concave side of each sheet of the hyperbolic cylinder (top of the upper sheet and bottom of the lower sheet). With no boundaries given, the input is:

SF	S	DV	NB			
5	7	3	0			
SSI						
-1.						
x_1	y_1	z_1	x_2	y_2	z_2	x_3
0.	0.	8.	0.	6.	10.	0.



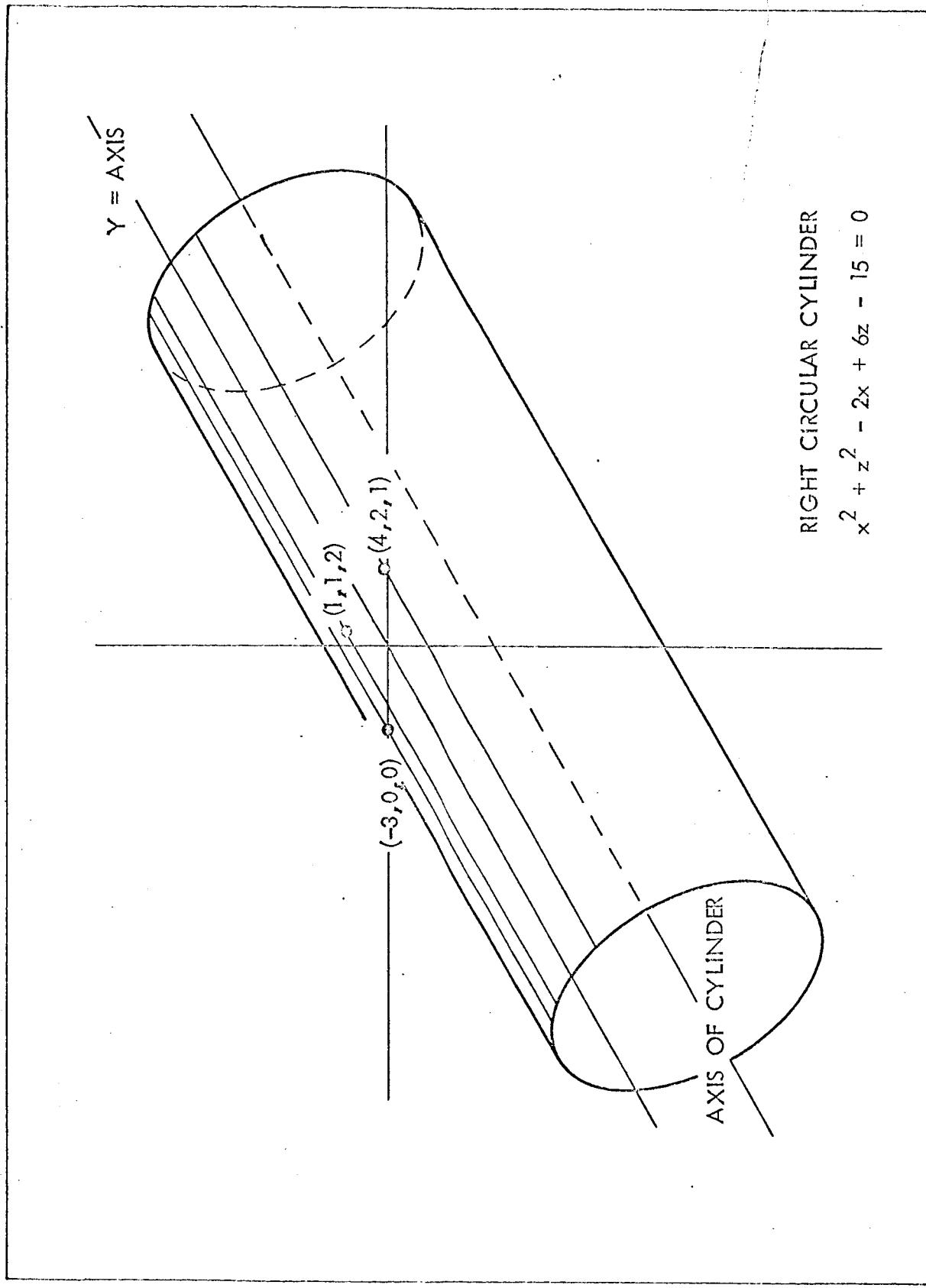


Figure 6-6. Example Problem #1 - Surface #6

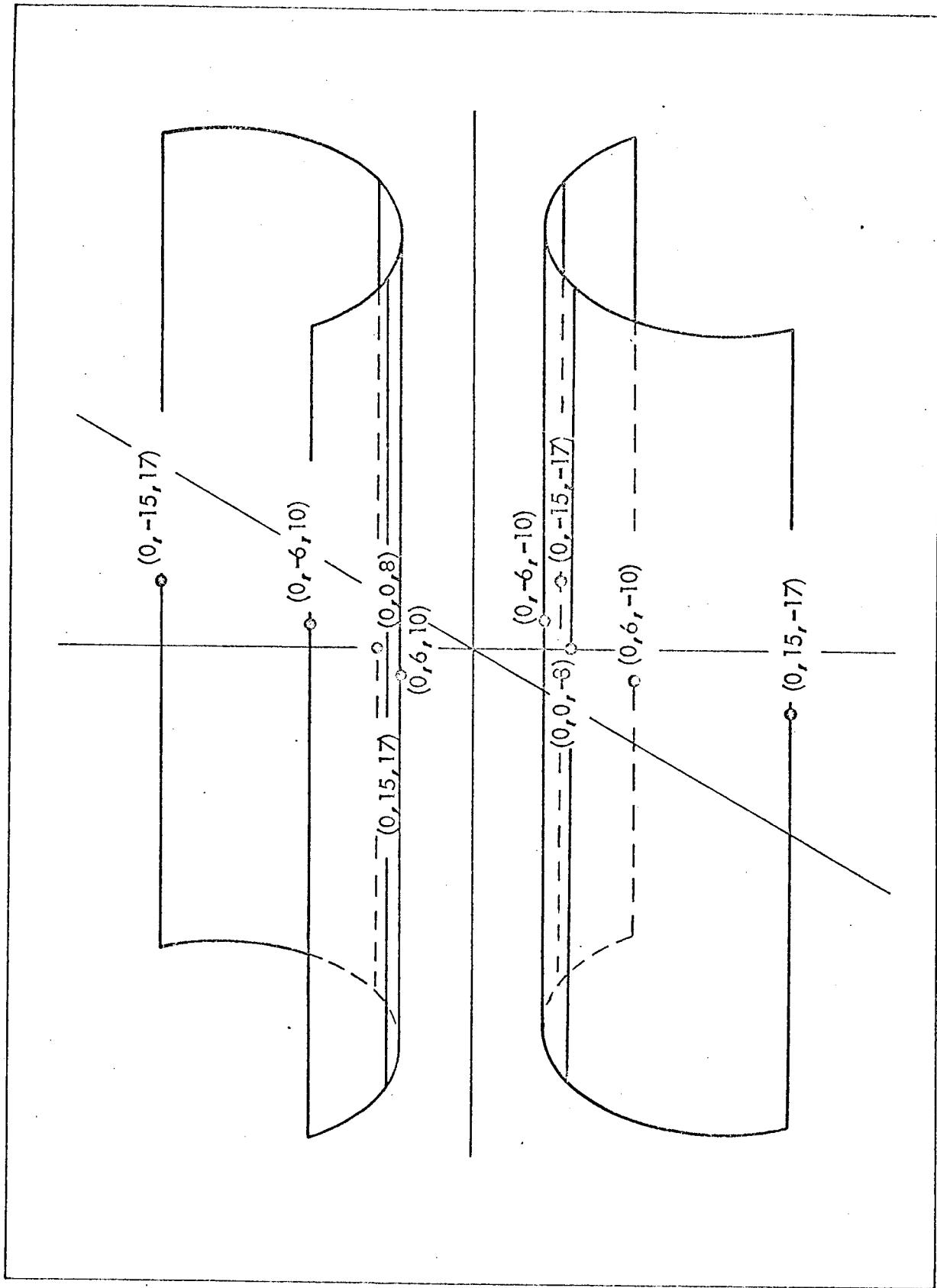


Figure 6-7. Example Problem #1 - Surface #7

y_3	z_3	x_4	y_4	z_4	x_5	y_5
-6.	10.	0.	15.	17.	0.	-15
z_5	L	M	N			
17.	1.	0.	0.			

Surface #8 - Quadric of Revolution - Surface #8 is the section of the paraboloid of revolution $x^2 + y^2 + 4z = 0$ cut by the sphere $x^2 + y^2 + z^2 - 1 = 0$ as shown in Figure 6-8. The input Flag 6 requires an axis point (0,0,0), three surface points (no two on the same cross-section) (0,0,0), (-2;2,-2), and (0,-2,-1), plus a second axis point (0,0,1). The dependent variable is z and one boundary is given. The reference point is (1,0,0) and the side index 1.0 gives the convex (outer) surface of the paraboloid. Since the area of surface #8 is required, the limits $-.97175 \leq x \leq .97175$ and $-.97175 \leq y \leq .97175$ and a value of 15 for GS are given. The input is:

SF	S	DV	NB			
6	8	3	1			
SSI	MIN-1	MAX-1	MIN-2	MAX-2	GS	
1.	-.97175	.97175	-.97175	.97175	15	
x_1	y_1	z_1	x_2	y_2	z_2	x_3
0.	0.	0.	0.	0.	0.	-2.
y_3	z_3	x_4	y_4	z_4	x_5	y_5
2.	-2.	0.	-2.	-1.	0.	0.
z_5						
1.						

Surface #9 - General Quadric - Surface #9 is the ellipsoid, Figure 6-9, $x^2 + 4y^2 + 9z^2 - 24x - 48y - 36z = 0$. Input Flag 7 and the nine surface points (0,0,0), (30,6,2), (-6,6,2), (12,15,2), (12,-3,2), (12,6,8), (12,6,-4), (0,0,4), and (24,0,4) define the surface. The dependent variable is y (Flag 2) and no boundaries are given. The ellipsoid contains the reference point (1,0,0) thus the side index 1.0, gives the inside. The input is:

SF	S	DV	NB
7	9	2	0

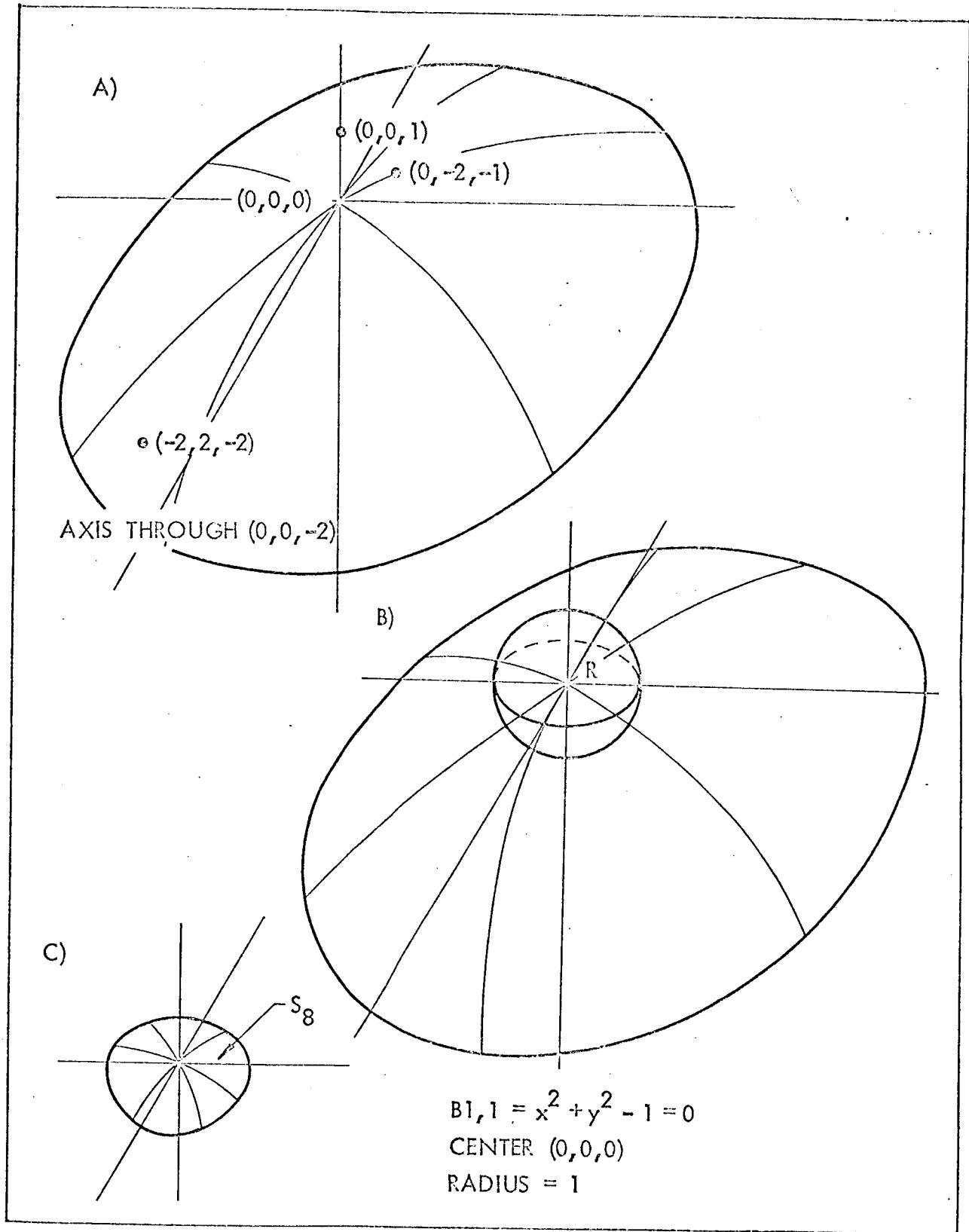


Figure 6-8. Example Problem #1 - Surface #8

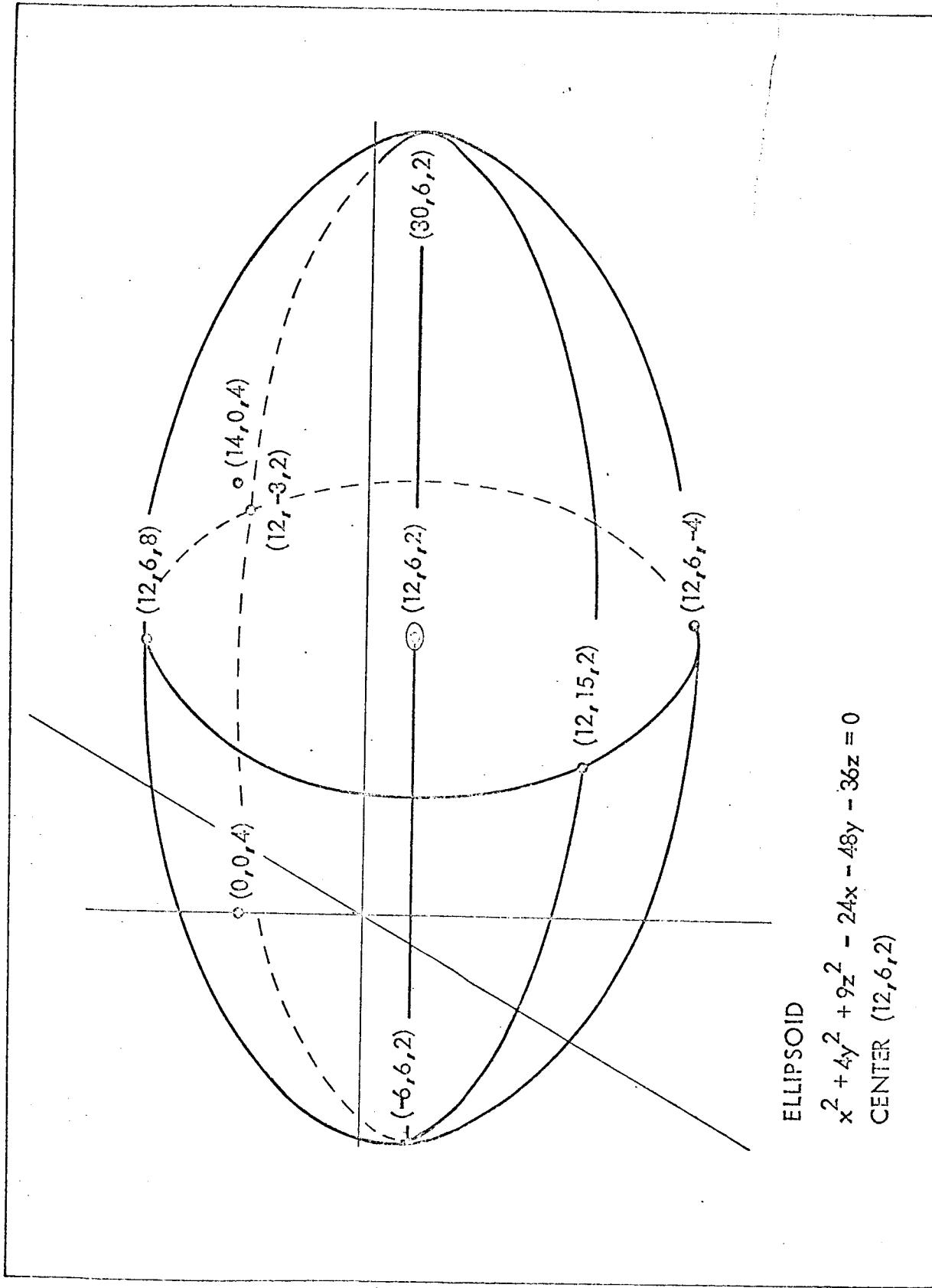


Figure 6-9. Example Problem #1 - Surface #9

SSI

1.

x_1	y_1	z_1	x_2	y_2	z_2	x_3
0.	0.	0.	30.	6.	2.	-6.
y_3	z_3	x_4	y_4	z_4	x_5	y_5
6.	2.	12.	15.	2.	12.	-3.
z_5	x_6	y_6	z_6	x_7	y_7	z_7
2.	12.	6.	8.	12.	6.	-4.
x_8	y_8	z_8	x_9	y_9	z_9	
0.	0.	4.	24.	0.	4.	

Surface #10 - Duplicate Surface Equation - Surface #10 (Figure 6-10) is a different section of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z - 35 = 0$ first defined in surface 4. Flag 8 and the surface number 4 define surface 10. The dependent variable for this surface is z and two boundaries are required. Side index -1.0 indicates the outside since the reference point (1,0,0) is inside the sphere. Note that the reference point is not printed out since this is a duplicated surface equation. The input is:

SF	S	DV	NB	NS
8	10	3	2	4 indicates that the equation of surface 4 is to be duplicated.

SSI

-1.

End of Surface Input - The end of the surface input block is marked by a card giving an input format ≥ 9 and a dummy surface number.

SF	S
9	11 A surface number not being used.

Boundary Input

Boundary Surface #1 - Listed Quadric Surface - The first boundary surface, labeled B(1,1) and meaning surface 1, boundary 1, is surface #3, labeled S(3), the sphere $x^2 + y^2 + z^2 - 1 = 0$. Since this boundary duplicates a surface, input Flag -1 is used with the number 3 of S(3). Since the reference point (0,0,0) which is not printed out for the duplicated equations of B(1,1) of S(3) and the required area of S(1) are both inside the sphere S(3),



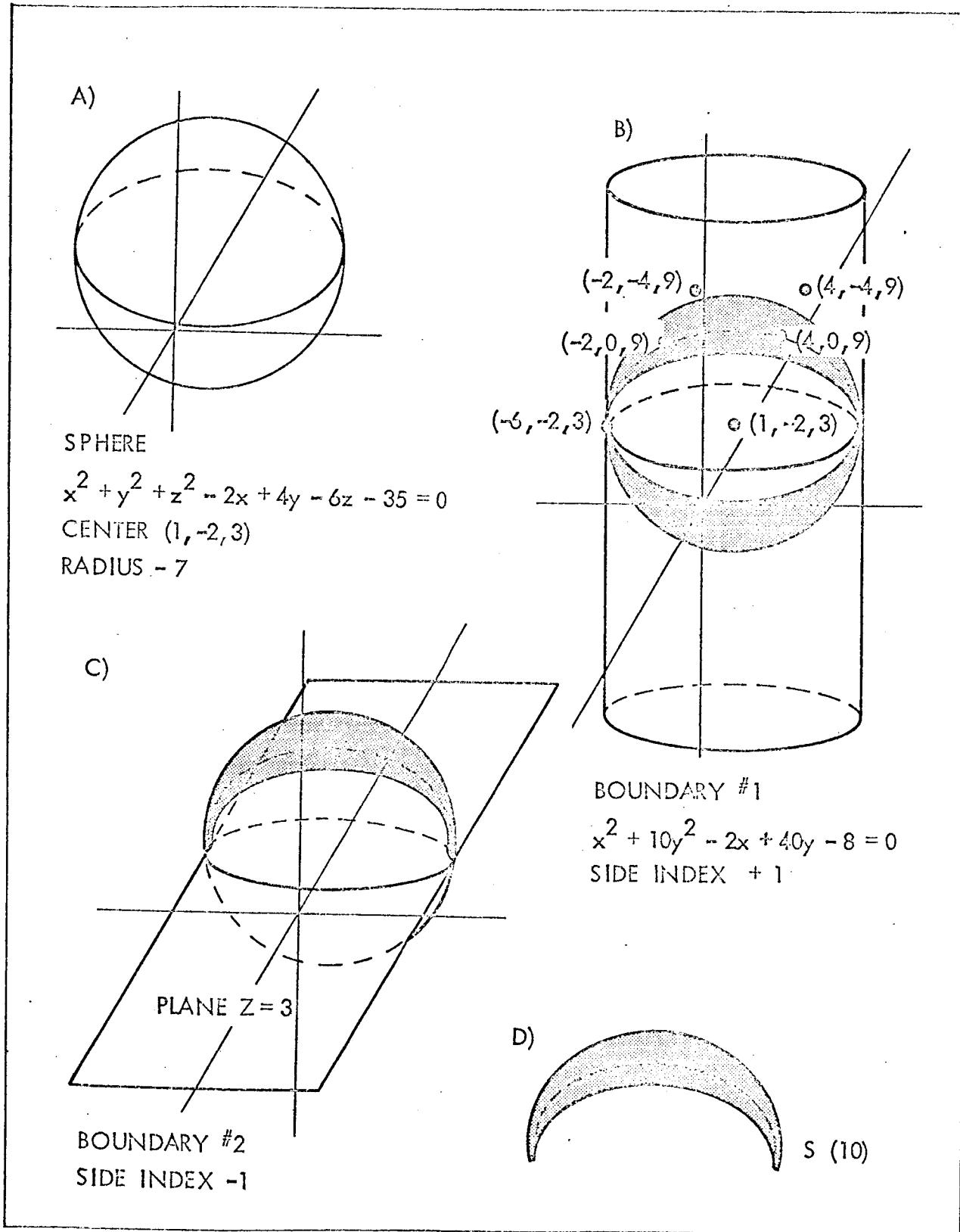


Figure 6-10. Example Problem #1 - Surface #10

a side index 1.0 is used. The input is:

SF	BSN	B	SN	BN	BSI
-1	1	1	3	--	1.

Boundary Surface #2 - Plane on a Plane - The second boundary surface, $B(2,1)$, is the first boundary on $S(2)$ and is the plane through the two points $(0,3,0)$ and $(0,0,-4)$, perpendicular to $S(2)$. Using input Flag 0, the surface is defined by the boundary points $(0,3,0)$ and $(0,0,-4)$. The reference point $(0,0,0)$ and the required area are both above the boundary plane hence the side index is 1.0. The input is:

BF	ESN	B	SN	BN	BSI
0	2	1	--	--	1.
x_1	y_1	z_1	x_2	y_2	z_2
0.	3.	0.	0.	0.	-4.

Boundary Surface #3 - Plane on Quadric Surface - The third boundary surface, $B(4,1)$, is the plane $x - 3z + 8 = 0$. With input Flag 1 the equation is defined by the three non-colinear boundary points $(1,-2,3)$, $(7,-5,5)$, and $(7,1,5)$. The required area and the reference point $(0,0,0)$ are below the plane giving a side index of 1.0. The input is:

BF	BSN	B	SN	BN	BSI
1	4	1	--	--	1.
x_1	y_1	z_1	x_2	y_2	z_2
1.	-2.	3.	7.	-5.	5.
y_3	z_3				7.
1.	5.				

Boundary Surface #4 - Right Circular Cylinder - The fourth boundary surface, $B(2,2)$, is the cylinder $x^2 + y^2 - 4 = 0$. With input Flag 2, the equation is defined by the two axis points $(0,0,0)$ and $(0,0,1)$, plus the radius $R = 2$. The reference point and the required area are both inside the cylinder, so the side index is 1.0. The input is:

BF	BSN	B	SN	BN	BSI
2	2	2	--	--	1.



x_1	y_1	z_1	x_2	y_2	z_2	R
0.	0.	0.	0.	0.	1.	2.

Boundary Surface #5 - Right Circular Cylinder - The fifth boundary surface, $B(4,2)$, is surface $y^2 + z^2 + 4y - 6z = 0$. This surface is defined by input Flag 3, the three boundary points $(7,0,0)$, $(7,1,5)$, and $(7,-5,5)$ (not all on the same element), plus the direction numbers $(1:0:0)$ of an element. The side index 1.0 gives the outside of the cylinder. The input is:

BF	BSN	B	SN	BN	BSI	
3	4	2	--	--	1.	
x_1	y_1	z_1	x_2	y_2	z_2	x_3
7.	0.	0.	7.	1.	5.	7.
y_3	z_3	L	M	N		
-5.	5.	1.	0.	0.		

Boundary Surface #6 - Quadric Cylinder - The sixth boundary surface, $B(10,1)$ is the elliptic cylinder $x^2 + 10y^2 - 2x + 40y - 8 = 0$. Input Flag 4, the five surface points $(-6,-2,3)$, $(-2,0,9)$, $(4,0,9)$, $(-2,-4,9)$, and $(4,-4,9)$ (no two on the same element), plus the direction numbers $(0 : 0 : 1)$ of an element. The inside of this cylinder is specified by the side index 1.0.

The input is:

BF	BSN	B	SN	BN	BSI	
4	10	1	--	--	1.	
x_1	y_1	z_1	x_2	y_2	z_2	x_3
-6.	-2.	3.	-2.	0.	9.	-2.
y_3	z_3	x_4	y_4	z_4	x_5	y_5
-4.	9.	4.	0.	9.	4.	-4.
z_5	L	M	N			
9.	0.	0.	1.			

Boundary Surface #7 - Quadric - The seventh boundary surface, $B(10,2)$, is the plane $z - 3 = 0$. Input Flag 5 and the ten coefficients A through J

of $z - 3 = 0$ are used to define this surface. The reference point $(0,0,0)$ is below the boundary plane; thus the side index -1.0 indicates the upper hemisphere of $S(10)$. The input is:

BF	BSN	B	SN	BN	BSI
5	10	2	--	--	-1.
A	B	C	D	E	F G
0.	0.	0.	0.	0.	0. 0.
H	I	J			
0.	1.	-3.			

Boundary Surface #8 - Duplicate Boundary Surface - The eighth boundary surface, $B(8,1)$, is $x^2 + y^2 + z^2 - 1 = 0$. Since this is the same as boundary surface $B(1,1)$, input Flag 6 and the surface number 1 plus the boundary number 1 define the boundary surface area. Side index 1.0 [(reference point for $B(8,1)$ = reference point for $B(1,1)$)] specifies the portion of $S(8)$ contained within the sphere. The input is:

BF	BSN	B	SN	BN	BSI
6	8	1	1	1	1.

End of Boundary Block - A card giving input Flag 7 with a dummy surface and boundary numbers is required to close the block.

BF	BSN	B
7	11	1

Computation Control

Grid Point Computation and Storage - Surfaces $S(1)$, $S(2)$ and $S(8)$ are used in shape factor or surface area computation. Therefore grid points must be computed and stored for these surfaces and the input is:

K	s(s)	s(t)
-1	No previously stored grid points	1 2
0	Previously stored $S(1)$ and $S(2)$ grid points are on tape	8

Surface-to-Surface Shape Factor Without Interference - The shape factor $F(1,2)$ from $S(1)$ to $S(2)$ is required with no interference. Therefore the input is:

K	s(s)	s(t)	NIS
1	1	2	0



Surface-to-Surface Shape Factor With Interference - To compute the shape factor $F(1,2)$ with interference by $S(8)$, the input is:

K	$S(s)$	$S(t)$	NIS
1	1	2	1
IS			
8			

Point-to-Surface Shape Factor Without Interference - To compute the point-to-surface shape factor $P(1)F(1,2)$ from point $P(1)$ with independent coordinates $(0.,0.)$ on $S(1)$ to $S(2)$ without interference requires the control Flag 4, the surface numbers 1 and 2, the dependent variable z on $S(1)$, the number of points, 1, the side index on $S(1)$, the independent coordinates $(0,0)$ of $P(1)$, and the number of interfering surfaces 0. This is given in the form:

K	$S(s)$	$S(t)$	NIS
2	1	2	0
DV	NP	SSI	
3	1	-1.	
x_1	y_1	(Since z is dependent)	
0.	0.		

Point-to-Surface Shape Factor With Interference - The input for this case is a combination of the inputs for the surface-to-surface shape factor with interference and the point-to-surface shape factor. The input for the shape factor $P(1)F(1,2)$ from point $P(1)$ on $S(1)$ with interference by $S(8)$ is:

K	$S(s)$	$S(t)$	NIS
2	1	2	1
DV	NP	SSI	
3	1	-1.	
x_1	y_1		
0.	0.		
IS			
8			

Surface Area - Control Flag 3 and the surface number are required to obtain any surface areas not computed in the surface-to-surface shape factor routine. The input for the areas of S(2) and S(8) is:

K	S(s)
3	2
K	S(s)
3	8

End of Input - The end of input is signaled by a card with control Flag 4.

K
4

Discussion of Problem

The input sheets required for this problem and the computed output are given in Tables 6-1 and 6-2, respectively. Since this example problem is intended only to give samples of each type of input and output, the values computed are not discussed.

In reading the output, point $Q = (x, y, z)$ appearing in the printout of the equations for surfaces $S(1)$ through $S(9)$ and all the boundary surfaces except $B(1,1)$ and $B(8,1)$ is the surface or boundary surface reference point. The numbers in the lines immediately below the reference point Q , which occur in the printout for surfaces $S(2)$, $S(4)$, $S(6)$, $S(7)$, $S(8)$, and $S(9)$, plus boundary surfaces $B(2,1)$, $B(4,1)$, $B(4,2)$, and $B(10,1)$, are the values obtained by substituting the coordinates of the surface points used to define the surface equation into the equation. If one or more of these values approaches the magnitude of the coefficients, the limited accuracy of the digital solution has probably resulted in an invalid equation. Note the machine discards the equation if any of these values exceeds 0.1. However, this inflexible test is not an absolute test, so these values should be compared with the size of the coefficients.

For the surface-to-surface shape factor, the first line headed by "DELS" gives the increments computed for the rectangle of the independent variables with the given GS. The second line gives the surface numbers s and t plus the product $A(s)F(s,t)$. The third line gives the surface number s and the area, $A(s)$, of $S(s)$. The last line in this output gives the surface numbers s and t plus the value $F(s,t)$.



TABLE 6-1
EXAMPLE #1 INPUT

SEQ.	GENERAL PURPOSE DATA SHEET										LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION										PAGE OF JOB NO.				
	PREPARED	NAME	DATE	TITLE																					
CHECKED													NO	EWA											GROUP
ID	10	20	30	40	50	60	65	70	72	73	76														
001	5	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
002	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
003	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
004	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
005	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
006	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
007	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
008	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
009	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
010	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
011	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
012	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
013	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
014	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
015	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
016	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
017	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
018	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
019	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
020	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
021	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
022	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
023	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
024	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		



TABLE 6-1 (Continued)

SEQ.	GENERAL PURPOSE DATA SHEET										LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION										PAGE <i>67</i> OF JOB NO.		
	PREPARED	NAME	DATE	TITLE	W.O.	EWA	ID																
CHECKED																							
77	80	1	10, 1	30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 76																			
026	-64		15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 76																				
027	17	1,	10, 1	15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 76																			
028	6	6,	9,	15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 76																			
029	1	-97175	97175	97175	15,																		
030	1	1,	9,	97175	15,																		
031	2	5,	12,	12,	15,																		
032	1	-2,	12,	12,	15,																		
033	7		9,	9,	12,																		
034	1		7,	7,	10,																		
035	0		5,	5,	8,																		
036	1		5,	5,	8,																		
037	2		12,	12,	15,																		
038	0		12,	12,	15,																		
039	1		9,	9,	12,																		
040	1		8,	8,	10,																		
041	1		9,	9,	11,																		
042			12,	12,	15,																		
043	0		12,	12,	15,																		
044	0		3,	3,	5,																		
045	1		1,	1,	3,																		
046	1		12,	12,	15,																		
047	1		5,	5,	8,																		
048	1		2,	2,	3,																		
049	0		0,	0,	2,																		
050	3		3,	3,	4,																		



TABLE 6-1 (continued)

SEQ.	GENERAL PURPOSE DATA SHEET			LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION												PAGE OF JOB NO.	
	PREPARED	NAME	DATE	TITLE													
	CHECKED			W.O.						EWA						GROUP	
ID				10	15	20	25	30	35	40	45	50	55	60	65	70	72 73 74
251	2501	5	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
252	251	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
253	252	4	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	4.
254	253	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
255	254	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
256	255	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
257	256	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
258	257	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
259	258	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
260	259	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
261	260	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
262	261	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
263	262	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
264	263	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
265	264	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
266	265	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
267	266	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
268	267	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
269	268	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
270	269	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
271	270	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
272	271	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
273	272	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
274	273	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
275	274	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.



TABLE 6-1 (Continued)



TABLE 6-2. OUTPUT FOR EXAMPLE #1

```

-1 1.000 1.000 3 1.000 -1.000 1.000 0 16.000
1.000 1.000 0 0.000 0.000 0.000 0.000 0.000 0.

C= 0.01 FOR SURFACE NO. 1

COEFFICIENTS FOR SURFACE NO. 1 A= 0. E= 0. B= 0. F= 0.
C= 0. D= 0. H= 0. I= 0. 0.09999999E 01 J= 0.
G= 0. D= 0. H= 0. I= 0. 0.09999999E 01 J= 0.
H= 0. D= 0. H= 0. I= 0. 0.09999999E 01 J= 0.

C= 0.01 FOR SURFACE NO. 2 A= 0. E= 0. B= 0. F= 0.
C= 0. D= 0. H= 0. I= 0. 0.25000000E 00 J= 0.09999999E
G= -0.50000000 00 H= -0.33333333E 00 I= 0.25000000E 00 J= 0.09999999E
H= 0.00000000 00 I= 0.00000000 00 J= 0.09999999E
I= 0.00000000 00 J= 0.09999999E

C= 0.01 FOR SURFACE NO. 3 A= -0.09999999E 01 B= -0.09999999E 01
C= -0.09999999E 01 D= -0. E= -0. F= -0.
G= 0. H= 0. I= 0. J= 0.09999999E
H= 0. I= 0. J= 0.09999999E
I= 0. J= 0.09999999E

C= 0.01 FOR SURFACE NO. 4 A= -0.2857142857E 01 B= -0.2857142857E 01
C= -0.2857142857E 01 D= -0. E= -0. F= -0.
G= -0.57142857E 01 H= -0.11428571E 00 I= 0.17142857E 00 J= 0.2857142857E 01
H= 0.57142857E 01 I= 0.00000000 00 J= 0.2857142857E 01
I= 0.00000000 00 J= 0.2857142857E 01

C= 0.01 FOR SURFACE NO. 5 A= 0.59652323E 07 B= 0.37227702E 07
C= 0.37227702E 07 D= 0.29202161E 07 E= -0.82825616E 07 F= 0.57652323E
G= -0.26843545E 09 H= 0.13421772E 09 I= 0.26843545E 08 J= 0.09999999E
H= 0.26843545E 09 I= 0.00000000 00 J= 0.09999999E
I= 0.00000000 00 J= 0.09999999E

C= 0.01 FOR SURFACE NO. 6 A= -0.66666667E 01 B= 0. F= 0.
C= -0.66666667E 01 D= 0. E= 0.
G= 0.13333333E 00 H= -0. I= -0.39999999E 00 J= 0.09999999E
H= 0.13333333E 00 I= 0.00000000 00 J= 0.09999999E

```



TABLE 6-2 (Continued)

```

-1.000 -0. -0. -0. -0. -0. -0.

0. 0. 0. 0. 0. 0. 0.

-6.000 10.000 10.000 15.000 6.000 10.000 0.

17.000 1.000 0. 0. 17.000 0. -15.000

Q=10.00 FOR SURFACE NO. 7
C=1.74505006E-09 C=1.49001161E-07 C=1.49001161E-07
COEFFICIENTS FOR SURFACE NO. 7 A= 0. E= 0. B= 0.17136335E-06
C= -0.15624599E-01 D= 0. F= 0.15624999E-01
G= -0. H= -0.76809351E-08 I= -0.92267822E-08 F= 0.74899851E-01
J= 0.05999999E 01

1.000 -0.978 0.973 -0.972 -0.972 0.972 15.000
A= 0. 0. 0. 0. 0. 0. -2.000
2.000 -2.000 0. -2.000 -1.000 0. 0.
1.000 0. 0. 0. 0. 0. 0.

Q=11.00 FOR SURFACE NO. 8
A= 0. B= 0. C= 0. D= 0. E= 0. F= 0. G= 0. H= 0. I= 0. J= 0.

COEFFICIENTS FOR SURFACE NO. 8 A= 0.09899999E 01 B= 0.09899999E 01
C= -0. D= -0. E= -0. F= -0. G= -0. H= -0. I= -0. J= -0.
G= -0. H= -0. I= -0. J= -0.

1.000 -0. -0. -0. -0. -0. -0. -0. -0. -0. -0.
A= 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
6.000 2.000 12.000 15.000 2.000 12.000 6.000 -6.000
2.000 1.0000 3.0000 8.0000 12.0000 6.0000 -3.0000
A= 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.

Q=11.00 FOR SURFACE NO. 9
A= -0.29000273E-05 B= -0.38743019E-05 C= -0.37252503E-05 D= -0.14305115E-05
C= -0.4040967E-06 E= -0.4173251E-05 F= -0.86426735E-06 G= -0.14305115E-05

COEFFICIENTS FOR SURFACE NO. 9 A= -0.4347865E-01 B= -0.17391306E-00 C= -0.1425328E-07 F= -0.10366026E-06
D= -0.10130637E-00 E= 0.1425328E-07 G= 0.15652178E 01 H= 0.
G= 0.10434734E 01 I= 0.15652178E 01 J= 0.

B= 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
I= 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
-1.000 -0. -0. -0. -0. -0. -0. -0. -0. -0.

COEFFICIENTS FOR SURFACE NO. 10 A= -0.28571424E-01 B= -0.28571424E-01 C= -0.28571424E-01 D= -0.28571424E-01
E= -0.36571424E-01 F= -0.36571424E-01 G= -0.36571424E-01 H= -0.36571424E-01 I= -0.36571424E-01 J= -0.36571424E-01
G= -0.57112852E-01 H= -0.11428571E-09 I= 0.17142057E-09 J= 0.09999999E 01
H= 0.57112852E-01 I= -0. 0. 0. 0. 0. 0. 0. 0. 0. 0.

```



TABLE 6-2 (Continued)

-1	1	1	1	3	0	1.000							
COEFFICIENTS FOR DRY NO. 1 OF SURFACE NO. 1 A= -0.09999999E 01 B= -0.09999999E 01	C= -0.09999999E 01 D= -0. H= 0.	E= 0. F= 0. G= 0.	I= 0. J= 0.09999999E 01	K= 1.000	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.	T= 0.
G= 0. 2. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	D= 0. 74205006E-08	H= 0.	I= 0.	J= 0.	K= 0.	L= 0.	M= 0.	N= 0.	P= 0.	Q= 0.	R= 0.	S= 0.	T= 0.
Q=(1.0,0) FOR DRY NO. 1 OF SURFACE NO. 2													
COEFFICIENTS FOR DRY NO. 1 OF SURFACE NO. 2 A= 0. B= 0. C= -0. D= 0. E= 0. F= 0. G= 0.34772222E-09 H= -0.33333333E-09 I= 1.000 J= 0.09999999E 01	K= 0.	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.	T= 0.	U= 0.	V= 0.	W= 0.
1.000 1.000 5.000 3.000 7.000 -5.000 5.000 7.000													
Q=(1.0,0) FOR DRY NO. 1 OF SURFACE NO. 4 A= 0. B= 0. C= 0.74505006E-08 D= 0.74505006E-08 E= 0. F= 0. G= 0.	H= 0.	I= 0.	J= 0.74505006E-08	K= 0.	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.	T= 0.
Q=(1.0,0) FOR DRY NO. 1 OF SURFACE NO. 4 A= 0. B= 0. C= -0.12300000E-00 D= -0. H= 0.	E= 0.	F= 0.	G= 0.	I= -0.37200000E-00	J= 0.09999999E 01	K= 0.	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.
1.0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	D= 2. 0. 2. 0. 2. 0. 0. 0. 0. 0. 0. 0. 0. 0.	H= 0.	I= 0.	J= 0.	K= 1.000	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.
Q=(1.0,0) FOR DRY NO. 2 OF SURFACE NO. 2													
COEFFICIENTS FOR DRY NO. 2 OF SURFACE NO. 2 A= -0.25000000E-00 B= -0.25000000E-00 C= 0. D= 0. E= 0. F= 0. G= 0.	H= 0.	I= 0.	J= 0.	K= 0.	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.	T= 0.
7.000 7.000 5.000 1.000 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	D= 3. 0. 4. 0. 5. 0. 0. 1.000 0.	H= 0.	I= 0.	J= 0.	K= 1.000	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.
Q=(1.0,0) FOR DRY NO. 2 OF SURFACE NO. 4 A= 0. B= 0.23841858E-06 C= 0.20000000E-00 D= 0. E= 0. F= 0. G= 0.	H= 0.	I= 0.	J= 0.	K= 0.	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.	T= 0.
1.0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	D= 4. 1. 0. 1. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0.	H= 0.	I= 0.	J= 0.	K= 1.000	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.
Q=(1.0,0) FOR DRY NO. 1 OF SURFACE NO. 10 A= 0. B= 0.12500000E-00 C= 0.13245681E-07 D= 0. E= 0. F= 0. G= 0.	H= 0.	I= 0.	J= 0.	K= 0.	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.	T= 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	D= 5. 2. 5. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	H= -0.50000000E 01	I= 0.	J= 0.	K= 0.	L= 0.	M= 0.	N= 0.	O= 0.	P= 0.	Q= 0.	R= 0.	S= 0.



TABLE 6-2 (Continued)

n_* 0. 0. 0. 0. 0. 0.

n_* 1.000 -1.000

$Q = (0, 0, 0)$ FOR ADRY NO. 2 OF SURFACE NO. 10

COEFFICIENTS FOR ADRY NO. 2 OF SURFACE NO. 10
 $D_x = -0.$
 $E_x = -0.$
 $F_x = -0.$
 $G_x = -0.$
 $H_x = -0.$
 $I_x = 1.$
 $J_x = 0.13333333E-00$

$A_x = -0.$
 $B_x = -0.$
 $C_x = -0.$
 $D_x = -0.$
 $E_x = -0.$
 $F_x = -0.$
 $G_x = -0.$
 $H_x = -0.$
 $I_x = 1.000$

COEFFICIENTS FOR ADRY NO. 1 OF SURFACE NO. 8
 $D_x = -0.$
 $E_x = -0.$
 $F_x = -0.$
 $G_x = -0.$
 $H_x = 0.$
 $I_x = 0.$
 $J_x = 0.$



TABLE 6-2 (Continued)

SURFACE NUMBER	1	1	2	-0	-0
WAS PUT ON TAPE					
SURFACE NUMBER	2	2	WAS PUT ON TAPE		
	0	0	-0	-0	-0
UNDERFLOW AT 71533 IN MQ					
UNDERFLOW AT 71535 IN MQ					
SURFACE NUMBER	8	8	WAS PUT ON TAPE		
	1	1	2	-0	-0
DELT					
S= 1	T= 2	A(S,T)=	0.12500000E-00	0.12500000E-00	0.25000000E-00
S= 1	AREA (S)=	0.32500000E 01			
S= 1	T= 2	F(S,T)=	0.34399627E-00	1	-0
S= 1 T= 2 F(S,T) WITH INTERFERENCE BY SURFACE NOS.					
DELT	0.12500000E-00	0.12500000E-00	0.12500000E-00	0.12500000E-00	0.25000000E-00
S= 1	T= 2	A(S,T)=	0.21719380E-00	0.21719380E-00	
S= 1	AREA (S)=	0.32500000E 01			
S= 1	T= 2	F(S,T)=	0.66028664E-01	0	-0
S= 1 T= 2 F(S,T) WITH INTERFERENCE BY SURFACE NOS.					
POINT ON S IS X(T)=	2	F(T)=	0.34892590E-00	0	-0
	1		V(T)=	0	
	2		Z(T)=	0	-0
S= 1 T= 2 F(S,T) WITH INTERFERENCE BY SURFACE NOS.					
I= 1 S= 1 T= 2 F(T)=	2	P(T)=	0.25431555E-00	0	-0
POINT ON S IS X(T)=	2	V(T)=	0	-0	
	3	Z(T)=	0	-0	-0



TABLE 6-2 (Continued)

S = 2	AREA (S) = 8	0.16434066F 02	-0	-0
S = 8	AREA (S) = 4	0.31401466F 01	-0	-0



For the point-to-surface shape factor $P(i)F(s,t)$, the first line gives the point number i , the surface numbers s and t , plus the shape factor. The second line gives the coordinates of $P(i)$ in the format: "POINT ON S IS X(I)=_____ Y(I)=_____ Z(I)=_____". If interfering surfaces are given, the interference printout described above is added in front of the first line.

The surface area printout is simply the surface number s and the area $A(s)$.

EXAMPLE PROBLEM #2

The second example problem approximates a typical rocket nozzle and storage sphere problem. Three surfaces are involved. The first is a frustum of a cone, two units high, with bases of .75 units radius and 1.25 units radius. The second is a cylinder with a radius of .75 units on a common axis with the cone starting at the small end of the cone and extending 1.75 units below the cone. The third surface is a sphere of radius 1/2 whose center is 2 units from the axis of the cone cylinder combination and .75 units below their junction. This geometry is illustrated in Figure 6-11. The shape factors from cone to sphere, from sphere to cylinder, and from cylinder to cone are computed. This means that the three areas and the three AF products are all computed. Thus division of each of the AF products by the corresponding surface areas yields the remaining three shape factors.

Surface Input

Cone - The first four surfaces in the input are all parts of the cone $256x^2 + 256y^2 - 16z^2 - 72z - 81 = 0$. Surfaces S(1) and S(2) give the required frustum of the cone while S(3) and S(4) each give one-half of the given area.

S(1) is defined by input Flag 6 for a quadric of revolution, the axis point (0,0,.75), the three surface points (0,.75,.75), (0,1,1.75), and (0,1.25,2.75), plus the axis point (0,0,2.75). The dependent variable for S(1) is y and one boundary is used. The reference point (0,0,0) is inside the cone, hence the convex (outer) surface requires a side index of -1.0. The limits on the independent variables are $-1.25 \leq x \leq 1.25$ and $.75 \leq z \leq 2.75$, while GS is set equal to 15.



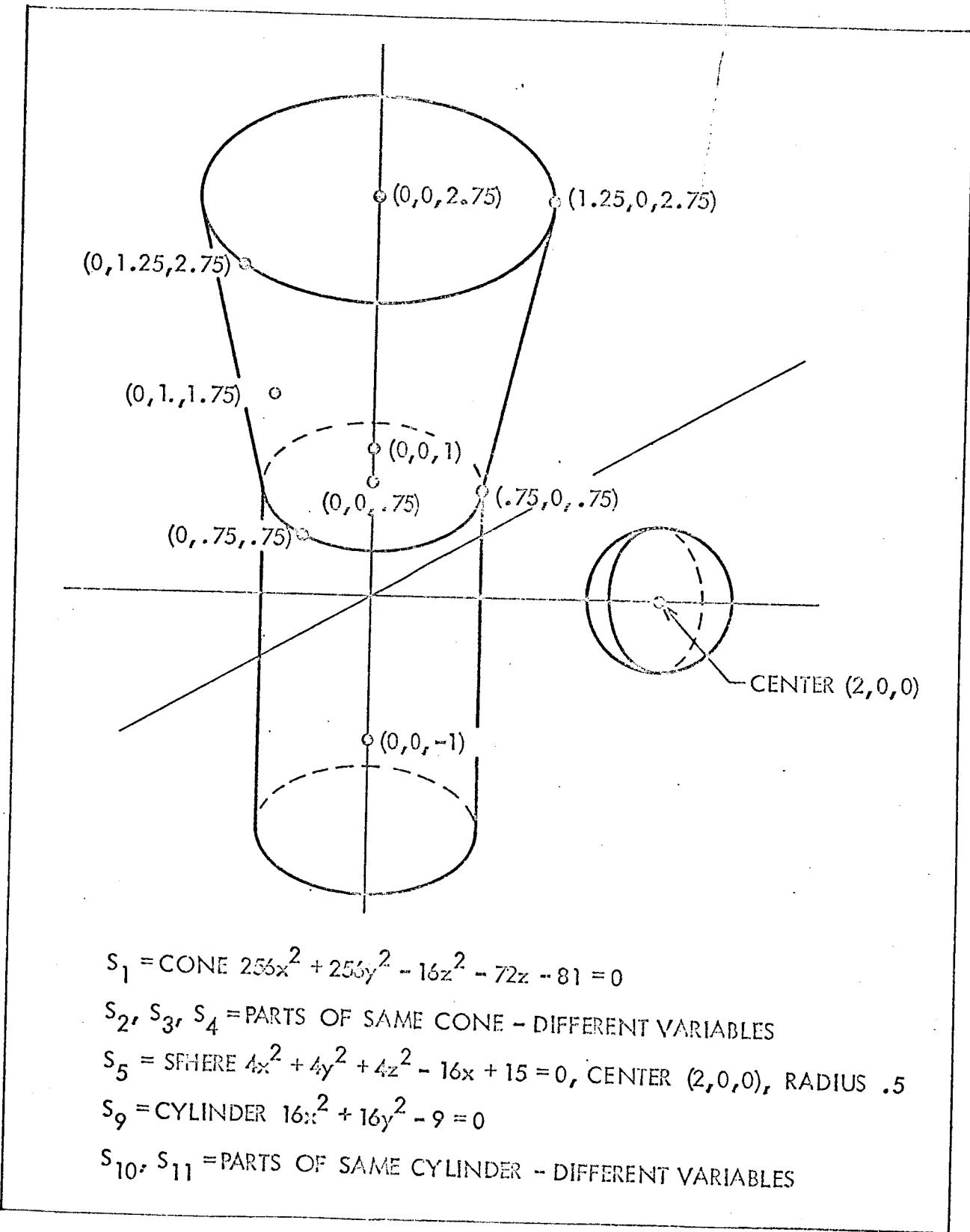


Figure 6-11. Example Problem #2

$S(2)$ is defined by input Flag 8 and the surface number 1 of $S(1)$. The dependent variable is z which changes the limits on the independent variables to $-1.25 \leq x \leq 1.25$ and $-1.25 \leq y \leq 1.25$. The two planes comprising the boundary $B(1,1)$ are separated and listed as two boundaries. The side index and GS remain unchanged.

$S(3)$ is defined by input Flag 8 and the surface number 1 of $S(1)$. The dependent variable on this half of the cone is x . Two boundaries are used, the new one being the perpendicular planes $x - y = 0$ and $x + y = 0$, which are combined in the single boundary surface $x^2 - y^2 = 0$. The surfaces $S(3)$ and $S(4)$, with this boundary, are shown in Figure 6-12. With x dependent and the given boundaries, the limits on the independent variables are $-1.25 \times \sqrt{2}/2 = -.8838838 \leq y \leq .8838838 = 1.25 \times \sqrt{2}/2$ and $.75 \leq z \leq 2.75$. The side index and GS remain unchanged.

$S(4)$ is defined by input Flag 8 and the surface number 1. The dependent variable on this half of the cone is y with limits $-.8838838 \leq x \leq .8838838$ and $.75 \leq z \leq 2.75$ on the independent variables. The number of boundaries, the side index and GS are unchanged.

Sphere - The next four surfaces all lie on the sphere $4x^2 + 4y^2 + 4z^2 - 16x + 15 = 0$. $S(5)$ gives the complete sphere while $S(6)$, $S(7)$, and $S(8)$ divide the sphere into approximately equal subsurfaces each with a different dependent variable (Figure 6-13).

$S(5)$ is defined by input Flag 1, the coordinates $(2,0,0)$ of the center, and the radius $R = .5$. The dependent variable is x and no boundaries are required. This sphere does not contain its reference point $(0,0,0)$, thus the side index 1.0 indicates the outside. The limits are $-.5 \leq y \leq .5$ and $-.5 \leq z \leq .5$ for the independent variables and GS is 16.

$S(6)$ is given by Flag 8 and the surface number 5. One boundary, the cone $x^2 - y^2 - z^2 - 4x + 4 = 0$, is used which cuts the limits on the independent variables to $-.5 \times \sqrt{2}/2 = -.3535535 \leq y \leq .3535535 = .5 \times \sqrt{2}/2$ and $-.3535535 \leq z \leq .3535535$. The dependent variable, side index, and GS are unchanged.

$S(7)$ is given by Flag 8 and the surface number 5. The dependent variable on $S(7)$ is y . Two boundaries, the cone $x^2 - y^2 - z^2 - 4x + 4 = 0$

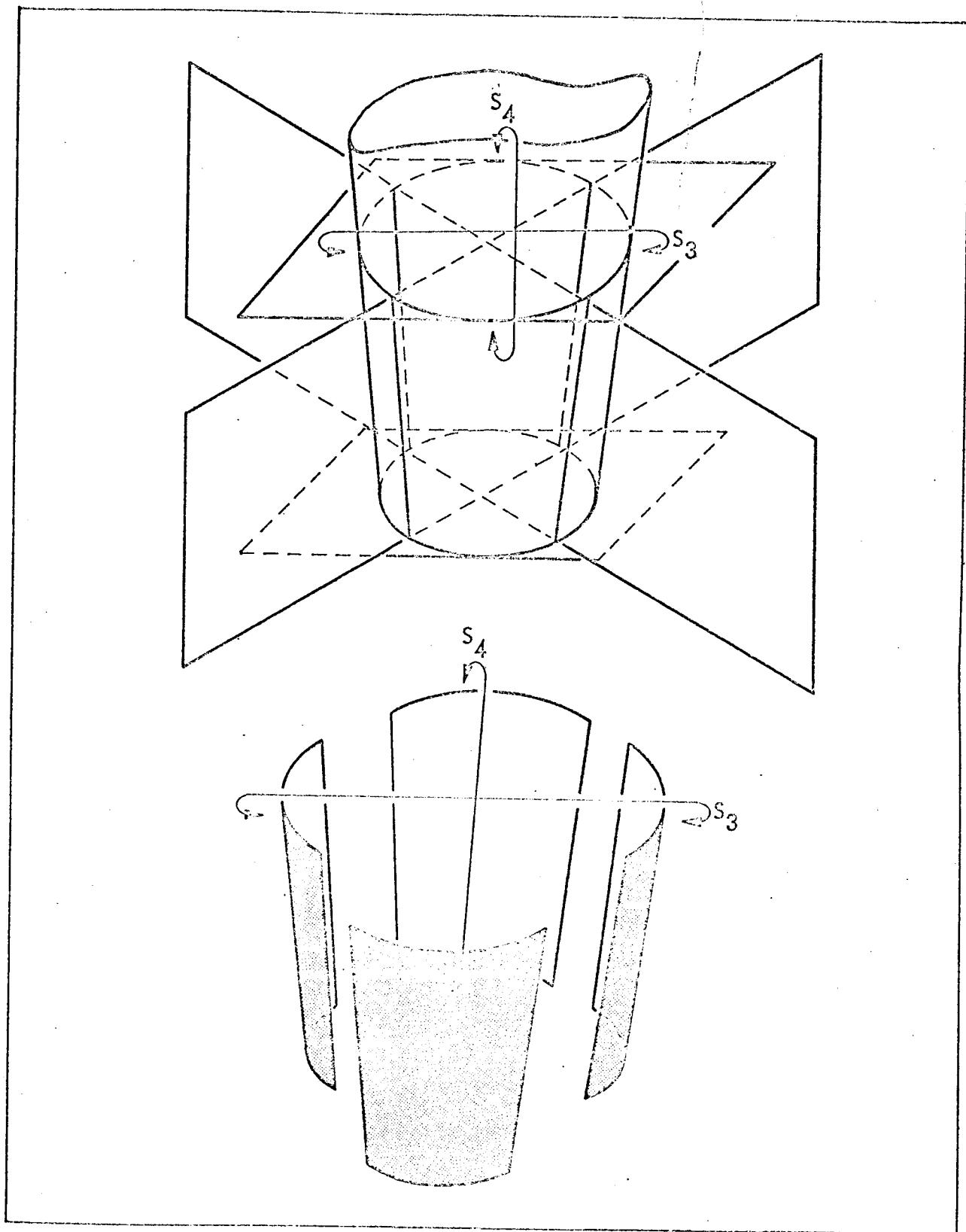


Figure 6-12. Example Problem #2 - Partitioned Cone

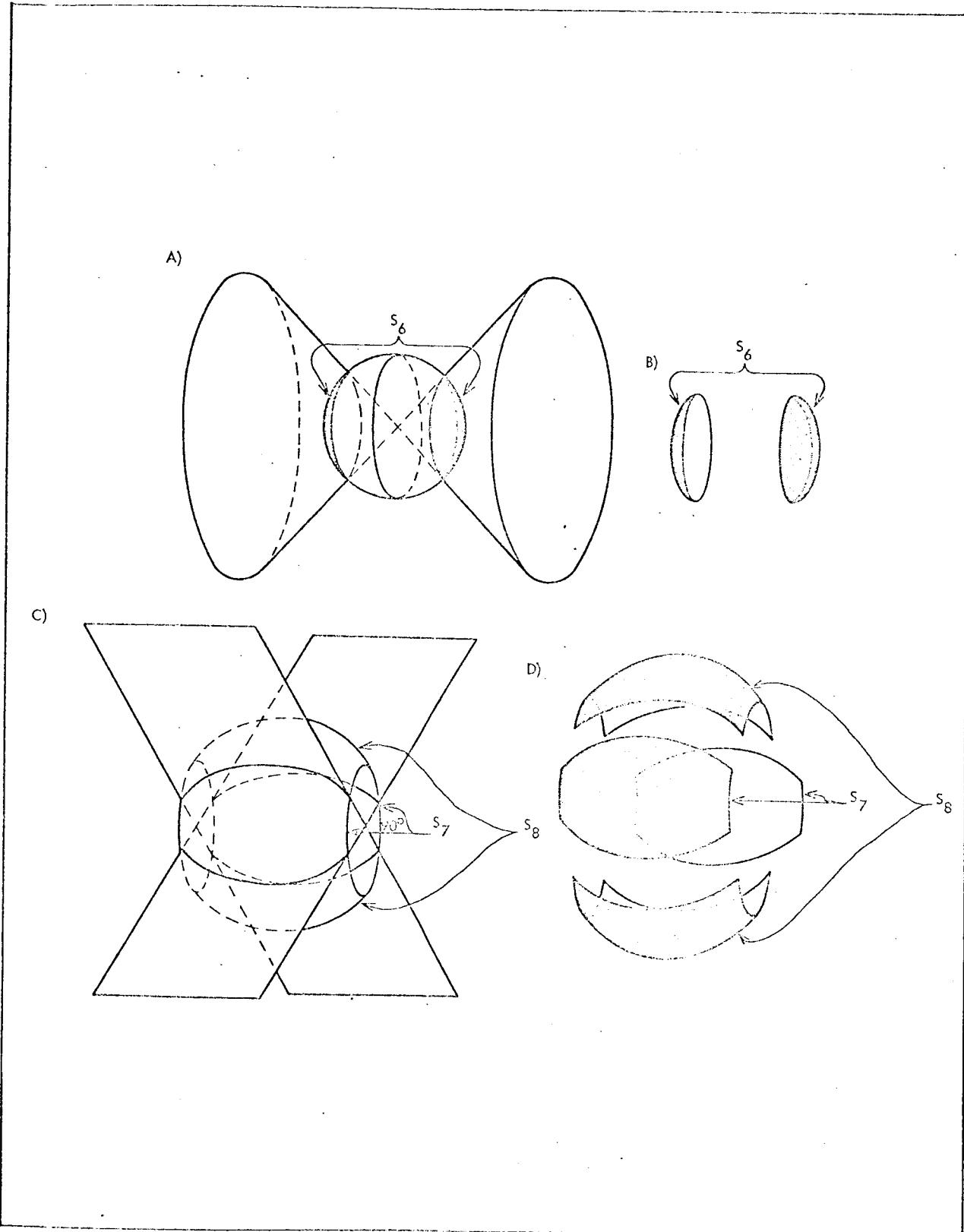


Figure 6-13. Example Problem #2 - Partitioned Sphere

and the surface $y^2 - z^2 = 0$ consisting of both the planes $y - z = 0$ and $y + z = 0$, are required. The limits on the independent variables are $2 - .3535535 = 1.646446 \leq x \leq 2.353554 = 2 + .5 x \sqrt{2}/2$ and $-.3535535 \leq z \leq .3535535$. The side index and GS are unchanged.

$S(8)$ is defined by input Flag 8 and the surface number 5. The dependent variable is z . The limits on the independent variables are $1.646446 \leq x \leq 2.353554$ and $-.3535535 \leq y \leq .3535535$. The number of boundaries, the side index, and GS are the same as for $S(7)$.

Cylinder - The last three surfaces all lie on the cylinder $16x^2 + 16y^2 - 9 = 0$. $S(9)$ gives the complete cylinder, while $S(10)$ and $S(11)$ each give one-half of the cylinder (Figure 6-14).

$S(9)$ is defined by input Flag 3, the axis points $(0,0,-1)$ and $(0,0,1)$, plus the radius $R = .75$. The dependent variable is x and one boundary is required. The reference point $(0,0,0)$ is inside the cylinder, so side index -1.0 is required for the outside. The limits are $-.75 \leq y \leq .75$ and $-1 \leq z \leq .75$ on the independent variables, while GS = 16.

$S(10)$ is given by Flag 8 and the surface number 9. Two boundaries are required, the new one being $x^2 - y^2 = 0$. The limits on the independent variables are $-.75 x \sqrt{2}/2 = -.5303303 \leq y \leq .5303303$ and $-1 \leq z \leq .75$. The dependent variable, side index and GS are unchanged.

$S(11)$ is given by Flag 8 and the surface number 9. The dependent variable is y and the limits are $-.5303303 \leq x \leq .5303303$ and $-1 \leq z \leq .75$ on the independent variables. The number of boundaries, the side index and GS are unchanged.

Boundary Input

Boundaries of the Cone - The frustum of the cone is bounded top and bottom by the surface $16z^2 - 56z + 33 = 0$, consisting of the two parallel planes $z - .75 = 0$ and $z - 2.75 = 0$. The subsections $S(3)$ and $S(4)$ (Figure 6-12) of the cone are separated by the surface $x^2 - y^2 = 0$, composed of the perpendicular planes $x - y = 0$ and $x + y = 0$.

Boundary $B(1,1)$, the surface $16z^2 - 56z + 33 = 0$, is defined by input Flag 5 and the ten coefficients A through J of the equation. The reference point $(0,0,0)$ for this boundary surface is below the lower surface $z - .75 = 0$.

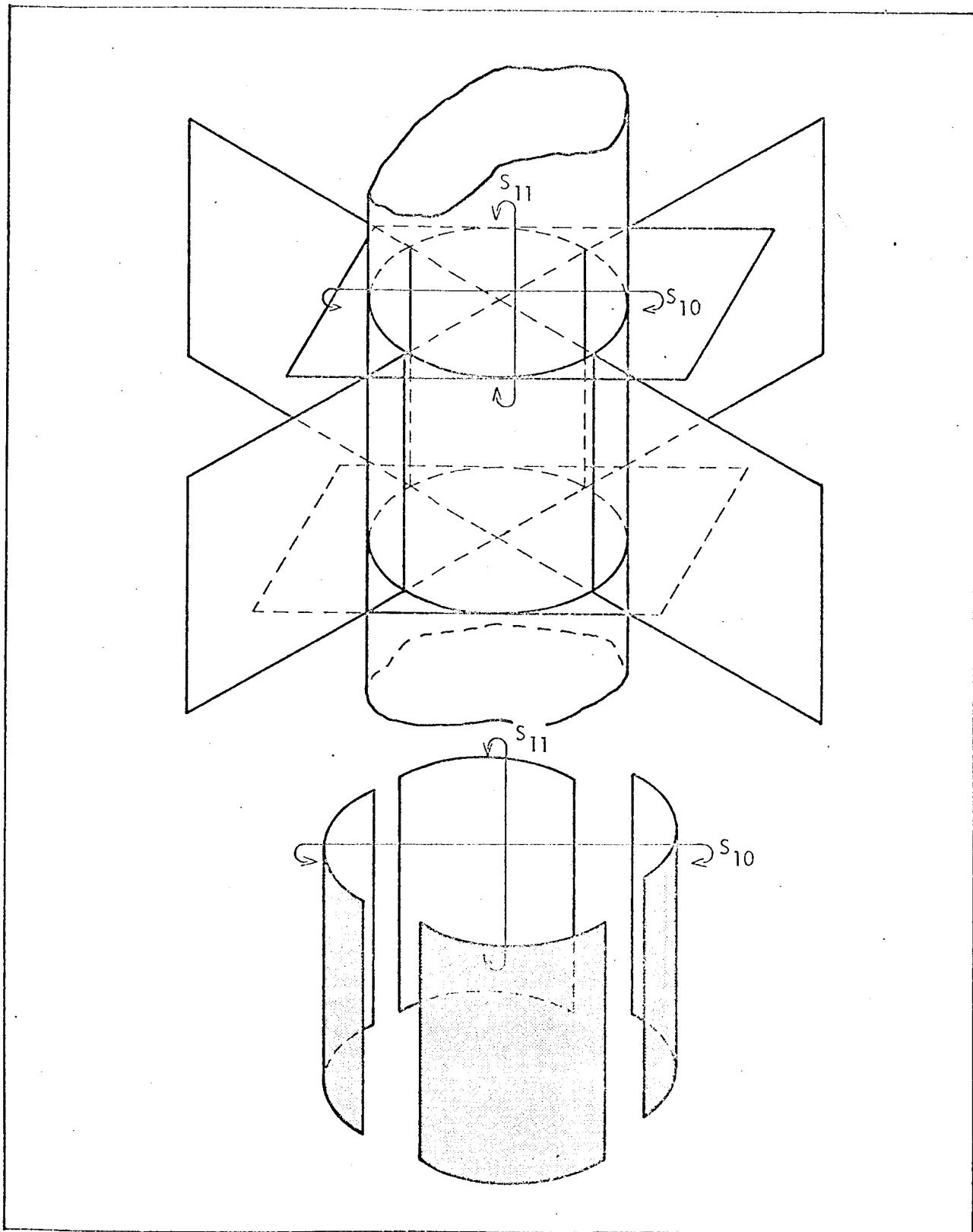


Figure 6-14. Example Problem #2 - Cylinder

while the required portion of the cone is between the planes $z - .75 = 0$ and $z - 2.75 = 0$. Therefore, the side index is -1.0.

$B(3,1)$ and $B(4,1)$ duplicate $B(1,1)$ and are given by input Flag 6 the surface number 1, and the boundary number 1. The side indicies remain -1.0.

$B(2,1)$ consists of the lower plane $z - .75 = 0$ of the surface $16z^2 - 56z + 33 = 0$. This boundary surface is defined by input Flag 1 and the three noncolinear points $(0,0,.75)$, $(0,.75,.75)$, and $(.75,0,.75)$ on the boundary surface.

The reference point $(0,0,0)$ is below the plane and the required portion of the cone is above it; therefore the side index is -1.0.

$B(2,2)$ is the upper plane $z - 2.75 = 0$. It is defined by input Flag 1 and the three non-colinear points $(0,0,2.75)$, $(0,1.25,2.75)$, and $(1.25,0,2.75)$ on the boundary plane. The reference point and the required surface area are both below $z - 2.75 = 0$, so the side index is 1.0.

$B(3,2)$ consists of the two perpendicular planes $x - y = 0$ and $x + y = 0$ combined in the surface $x^2 - y^2 = 0$. This boundary surface is defined by input Flag 5 and the 10 coefficients A through J of the equation $x^2 - y^2 = 0$. The reference point for boundary surface $B(3,2)$ is $(1,0,0)$ and the side index 1.0 specifies the two opposed sections of the cone required for $S(3)$.

$B(4,2)$ is the same as $B(3,2)$, so it is defined by input Flag 6, the surface number 3, and the boundary surface number 2. The remaining two sections of the cone are represented by $S(4)$ and are defined by a side index -1.0.

Boundaries on the Sphere - The three subsections $S(6)$, $S(7)$ and $S(8)$ of the sphere are separated by two boundary surfaces. The cone $x^2 - y^2 - z^2 - 4x + 4 = 0$ divides the sphere into three vertical slices. The two outside slices (Figure 6-13) form $S(6)$. The perpendicular planes $y - z = 0$, $y + z = 0$, given by the single surface $y^2 - z^2 = 0$, separate the center slice into four equal parts, two opposed sections apiece for $S(7)$ and $S(8)$.

$B(6,1)$, the cone $x^2 - y^2 - z^2 - 4x + 4 = 0$, is defined by the input Flag 5 and the ten coefficients A through J of the equation. The side index 1.0 and the reference point $(0,0,0)$ define the two end slices of the sphere.

$B(7,1)$ and $B(8,1)$ are the same cone defined by input Flag 6, the surface number 6 and the boundary surface number 1. On these two surfaces,

the center slice of the sphere is required so the side index is -1.0

$B(7,2)$ is the surface $y^2 - z^2 = 0$ which is defined by input Flag 5 and the 10 coefficients A through J of the equation. The side index for the two sections of $S(7)$ is 1.0.

$B(8,2)$ duplicates $B(7,2)$, with input Flag 6, the surface number 7, and the boundary surface number 2. The side index for $S(8)$ is -1.0.

Boundaries on the Cylinder - The cylinder, surfaces $S(9)$, $S(10)$, and $S(11)$, are bounded on the top and bottom by the surface $4z^2 + z - 3 = 0$ composed of the parallel planes $z + 1 = 0$ and $z - .75 = 0$. The two sections of the cylinder $S(10)$ and $S(11)$ are separated by the same surface, $x^2 - y^2 = 0$, that separates the two halves $S(3)$ and $S(4)$ of the cone (Figure 6-14).

$B(9,1)$ is the surface $4z^2 + z - 3 = 0$ which is defined by the input Flag 5 and the ten coefficients of $4z^2 + z - 3 = 0$. The required area of the cylinder and the reference point $(0,0,0)$ both lie between the boundary planes $z + 1 = 0$ and $z - .75 = 0$. Therefore, the side index is 1.0.

$B(10,1)$ and $B(11,1)$ duplicate $B(9,1)$, using input Flag 6, the surface number 9, and the boundary surface number 1.

$B(10,2)$, the surface $x^2 - y^2 = 0$, is defined as a quadric cylinder by input Flag 4, the five surface points $(0,0,0)$, $(1,1,0)$, $(2,2,0)$, $(1,-1,0)$, and $(-2,2,0)$, no two on the same element, plus the direction numbers $(0 : 0 : 1)$ of an element. The two sections of $S(10)$ are given by the side index 1.0.

$B(11,2)$ duplicates $B(10,2)$ with input Flag 6, the surface number 10, and the boundary number 2. As the two remaining sections (not in $S(10)$) are required for $S(11)$; the side index is -1.0.

Computation Control

Grid points are required for all eleven surfaces. The shape factors $F(2,5)$, $F(9,2)$, and $F(5,9)$ are required for the complete surfaces. The equivalent shape factors $F(3,6)$, $F(3,7)$, $F(3,8)$, $F(4,6)$, $F(4,7)$, $F(4,8)$, $F(6,10)$, $F(6,11)$, $F(7,10)$, $F(7,11)$, $F(8,10)$, $F(8,11)$, $F(10,3)$, and $F(10,4)$ in terms of the subsurfaces are also computed for comparison. Note that by symmetry $F(11,3)$ equals $F(10,4)$ and $F(11,4)$ equals $F(10,3)$, so those are not computed.

Discussion of the Problem

The input sheets for this problem and the output are found in Tables 6-3 and 6-4, respectively.

The relative accuracy of the computed values of the shape factors and the shape factor area products are evaluated in terms of the accuracy of the area computations.

The areas $A(1)$ or $S(1)$ and $A(11)$ or $S(11)$ are also computed for comparison.

The values for the area of the cone give the most striking example of the effect of the choice of dependent variables on the computed areas. The area on $S(1)$ is computed as 238, while the area on $S(2)$ is 12.4 and the sum of the areas on $S(3)$ and $S(4)$ is 12.7. Since the area of the frustum of a cone can be computed exactly, these numbers can be compared with the actual area. The area of the cone to one decimal is 12.9. Thus, the astronomical area computed for $S(1)$ with y -dependent is in error by a factor of slightly over 18. With z -dependent, the area of $S(2)$ shows an error of 3.9% while the division of the cylinder into two sections $S(3)$ with x -dependent and $S(4)$ with y -dependent reduces the error to 1.5%. Similarly, the area on the sphere $S(5)$ is 3.30, the area on the partitioned sphere 3.22 while the actual area is 3.14. For the cylinder $S(9)$, the area is 7.46 but the partitioned cylinder gives the exact (to two decimals) area 8.24.

Since the AF product is an additive quantity, the value of this product and hence the shape factor can be obtained for the partitioned surfaces by adding AF products, adding areas and then dividing the total AF product by the total area. For example, in computing the AF product from the cone to the sphere, the total AF product is obtained by adding $A(3)F(3,6)$, $A(3)F(3,7)$, $A(3)F(3,8)$, $A(4)F(4,6)$, $A(4)F(4,7)$, and $A(4)F(4,8)$. This is divided by the total area of the cone which is given by the sum of $A(3)$ and $A(4)$ to give the shape factor from cone to sphere. Note that the shape factors themselves are not additive.

The AF products from cone to sphere are .124 with single dependent variable or .132 for the partitioned surfaces. The AF product from the cylinder to cone shows more variation as would be expected since the points on the lines



TABLE 6-3
EXAMPLE #2 INPUT

SEQ.	NAME	DATE	GENERAL PURPOSE DATA SHEET												NO.	EWA	PAGE OF	JOB NO.	GROUP
			PREPARED	CHECKED	TITLE	10	15	20	25	30	35	40	45	50	55	60	65	70	75
221																			
222																			
223																			
224																			
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338																			
339																			

TABLE 6-3 (Continued)

SEQ.	GENERAL PURPOSE DATA SHEET				LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION	PAGE	OF
	PREPARED	NAME	DATE	TITLE			
	CHECKED						
	ID						
036	1	1	1	1	60	65	70
037	-1	-1	-1	-1	60	65	70
038	9	53033203	1/15	1/15	60	65	70
039	5	1/15	1/15	1/15	60	65	70
040	1	1	1	1	60	65	70
041	0	0	0	0	60	65	70
042	1	1	1	1	60	65	70
043	0	0	0	0	60	65	70
044	0	0	0	0	60	65	70
045	1	1	1	1	60	65	70
046	0	0	0	0	60	65	70
047	0	0	0	0	60	65	70
048	0	0	0	0	60	65	70
049	0	0	0	0	60	65	70
050	0	0	0	0	60	65	70



TABLE 6-3 (Continued)

GENERAL PURPOSE DATA SHEET				LOCKHEED-CALIFORNIA COMPANY, A DIVISION OF LOCKHEED AIRCRAFT CORPORATION												PAGE	OF
PREPARED	NAME	DATE	TITLE													JOB NO.	
CHECKED				XO												EWA	GROUP
SEQ.	ID			30	35	40	45	50	55	60	65	70	75	80	85	90	95
77	80	1		/	/	/	/	/	/	/	/	/	/	/	/	/	/
251				6	6	6	6	6	6	6	6	6	6	6	6	6	6
252				6	6	6	6	6	6	6	6	6	6	6	6	6	6
253				5	5	5	5	5	5	5	5	5	5	5	5	5	5
254	0	1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
255	0	1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
256				5	5	5	5	5	5	5	5	5	5	5	5	5	5
257				4	4	4	4	4	4	4	4	4	4	4	4	4	4
258	0	1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
259	2	1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
260	0	1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
261				5	5	5	5	5	5	5	5	5	5	5	5	5	5
262				4	4	4	4	4	4	4	4	4	4	4	4	4	4
263				3	3	3	3	3	3	3	3	3	3	3	3	3	3
264				2	2	2	2	2	2	2	2	2	2	2	2	2	2
265				1	1	1	1	1	1	1	1	1	1	1	1	1	1
266				0	0	0	0	0	0	0	0	0	0	0	0	0	0
267				0	0	0	0	0	0	0	0	0	0	0	0	0	0
268				0	0	0	0	0	0	0	0	0	0	0	0	0	0
269				0	0	0	0	0	0	0	0	0	0	0	0	0	0
270				0	0	0	0	0	0	0	0	0	0	0	0	0	0
271				5	5	5	5	5	5	5	5	5	5	5	5	5	5
272				4	4	4	4	4	4	4	4	4	4	4	4	4	4
273				3	3	3	3	3	3	3	3	3	3	3	3	3	3
274				2	2	2	2	2	2	2	2	2	2	2	2	2	2
275				1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE 6-3 (Continued)

S.R.O.	GENERAL PURPOSE DATA SHEET			LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION										PAGE OF		
	PREPARED	NAME	DATE	TITLE										JOB NO.		
	CHECKED			NO	EWK										GROUP	
77-40				77	70	65	60	55	50	45	40	35	30	25	20	15
221				/	/	/	/	/	/	/	/	/	/	/	/	/
222				/	/	/	/	/	/	/	/	/	/	/	/	/
019				/	/	/	/	/	/	/	/	/	/	/	/	/
219				/	/	/	/	/	/	/	/	/	/	/	/	/
260				/	/	/	/	/	/	/	/	/	/	/	/	/
261				/	/	/	/	/	/	/	/	/	/	/	/	/
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268				/	/	/	/	/	/	/	/	/	/	/	/	/
269				/	/	/	/	/	/	/	/	/	/	/	/	/
270				/	/	/	/	/	/	/	/	/	/	/	/	/
271				/	/	/	/	/	/	/	/	/	/	/	/	/
272				/	/	/	/	/	/	/	/	/	/	/	/	/
273				/	/	/	/	/	/	/	/	/	/	/	/	/



TABLE 6-4. OUTPUT FOR EXAMPLE #2

Q=(0,0,0) FOR SURFACE NO. 1									
-2.14901161E-07									-0.20861626E-06
COEFFICIENTS FOR SURFACE NO. 1 A= -0.31604939E 01 B= -0.31604939E 01									
C= 0.19751042E-02	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.88888900E 00	J= 0.09999999E 01		
G= 0.	H= 0.	I= 0.	J= 0.						
A= 1.250	B= 1.250	C= 1.250	D= 1.250	E= 1.250	F= 1.250	G= 1.250	H= 1.250	I= 1.250	J= 1.250
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
COEFFICIENTS FOR SURFACE NO. 2 A= -0.31604939E 01 B= -0.31604939E 01									
C= 0.19753042E-02	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.88888900E 00	J= 0.09999999E 01		
G= 0.	H= 0.	I= 0.	J= 0.						
A= 0.894	B= 0.894	C= 0.894	D= 0.894	E= 0.894	F= 0.894	G= 0.894	H= 0.894	I= 0.894	J= 0.894
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
COEFFICIENTS FOR SURFACE NO. 3 A= -0.31604939E 01 B= -0.31604939E 01									
C= 0.19753042E-02	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.88888900E 00	J= 0.09999999E 01		
G= 0.	H= 0.	I= 0.	J= 0.						
A= 0.894	B= 0.894	C= 0.894	D= 0.894	E= 0.894	F= 0.894	G= 0.894	H= 0.894	I= 0.894	J= 0.894
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
COEFFICIENTS FOR SURFACE NO. 4 A= -0.31604939E 01 B= -0.31604939E 01									
C= 0.19753042E-02	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.88888900E 00	J= 0.09999999E 01		
G= 0.	H= 0.	I= 0.	J= 0.						
A= 0.500	B= 0.500	C= 0.500	D= 0.500	E= 0.500	F= 0.500	G= 0.500	H= 0.500	I= 0.500	J= 0.500
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Q=(0,0,0) FOR SURFACE NO. 5									
COEFFICIENTS FOR SURFACE NO. 5 A= 0.26666667E-00 B= 0.26666667E-00									
C= 0.26666667E-00	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.	J= 0.		
G= -0.10666666E 01	H= 0.	I= 0.	J= 0.						
A= 0.354	B= 0.354	C= 0.354	D= 0.354	E= 0.354	F= 0.354	G= 0.354	H= 0.354	I= 0.354	J= 0.354
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
COEFFICIENTS FOR SURFACE NO. 6 A= 0.26666667E-00 B= 0.26666667E-00									
C= 0.26666667E-00	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.	J= 0.		
G= -0.10666666E 01	H= 0.	I= 0.	J= 0.						
A= 0.354	B= 0.354	C= 0.354	D= 0.354	E= 0.354	F= 0.354	G= 0.354	H= 0.354	I= 0.354	J= 0.354
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
COEFFICIENTS FOR SURFACE NO. 7 A= 0.26666667E-00 B= 0.26666667E-00									
C= 0.26666667E-00	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.	J= 0.		
G= -0.10666666E 01	H= 0.	I= 0.	J= 0.						
A= 0.354	B= 0.354	C= 0.354	D= 0.354	E= 0.354	F= 0.354	G= 0.354	H= 0.354	I= 0.354	J= 0.354
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
COEFFICIENTS FOR SURFACE NO. 8 A= 0.26666667E-00 B= 0.26666667E-00									
C= 0.26666667E-00	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.	J= 0.		
G= -0.10666666E 01	H= 0.	I= 0.	J= 0.						
A= 0.354	B= 0.354	C= 0.354	D= 0.354	E= 0.354	F= 0.354	G= 0.354	H= 0.354	I= 0.354	J= 0.354
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
COEFFICIENTS FOR SURFACE NO. 9 A= 0.26666667E-00 B= 0.26666667E-00									
C= 0.26666667E-00	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.	J= 0.		
G= -0.10666666E 01	H= 0.	I= 0.	J= 0.						
A= 0.354	B= 0.354	C= 0.354	D= 0.354	E= 0.354	F= 0.354	G= 0.354	H= 0.354	I= 0.354	J= 0.354
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
COEFFICIENTS FOR SURFACE NO. 10 A= 0.26666667E-00 B= 0.26666667E-00									
C= 0.26666667E-00	D= 0.	E= 0.	F= 0.	G= 0.	H= 0.	I= 0.	J= 0.		
G= -0.10666666E 01	H= 0.	I= 0.	J= 0.						
A= 0.354	B= 0.354	C= 0.354	D= 0.354	E= 0.354	F= 0.354	G= 0.354	H= 0.354	I= 0.354	J= 0.354
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Q=(0,0,0) FOR SURFACE NO. 9									

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TABLE 6-4 (Continued)

COEFFICIENTS FOR SURFACE NO. 9					
C=	0.	D=	0.	A=	-0.1777777E 01
G=	0.	H=	-0.	F=	0.
B		I=	-0.	J=	0.09999999E 01
-1.000	-0.530	0.530	-1.000	0.750	16.000
COEFFICIENTS FOR SURFACE NO. 10					
C=	0.	D=	0.	A=	-0.17777777E 01
G=	0.	H=	-0.	F=	0.
B		I=	-0.	J=	0.09999999E 01
-1.000	-0.530	0.530	-1.000	0.750	16.000
COEFFICIENTS FOR SURFACE NO. 11					
C=	0.	D=	0.	A=	-0.17777777E 01
G=	0.	H=	-0.	F=	0.
B		I=	-0.	J=	0.09999999E 01
9	12	-0	-0	-0	



TABLE 6-4 (Continued)

	5	1	1	0	0	-1.000					
	6.	0.	15.000	0.	0.	0.					
	6.	-56.000	33.000								
Q=10,0,0)	FOR BODY NO. 1	OF SURFACE NO. 1									
Coefficients for BODY NO. 1 OF SURFACE NO. 1	A= 0.	E= 0.	I= -0.16969696E 01	F= 0.	B= 0.						
C= 0.48649494E-00 D= 0.	H= 0.	T= 0.	J= 0.09999999E 01								
G= 1.	2.	1.	0.								
	0.	0.	0.750	0.	0.750	0.750	0.				
	0.	0.750									
Q=10,0,0) FOR BODY NO. 1	OF SURFACE NO. 2										
Coefficients for BODY NO. 1 OF SURFACE NO. 2	A= 0.	E= 0.74505806E-03	I= 0.74505806E-03	F= 0.	B= 0.						
Coefficients for BODY NO. 1 OF SURFACE NO. 2	A= 0.	E= 0.	I= 0.	F= 0.	B= 0.						
C= 0.	D= 0.	H= 0.	T= 0.	J= 0.09999999E 01							
G= 0.	2.	0.	0.								
	0.	0.	2.750	0.	1.250	2.750	1.250				
Q=10,0,0) FOR BODY NO. 2	OF SURFACE NO. 2										
Coefficients for BODY NO. 2 OF SURFACE NO. 2	A= 0.	E= 0.74505806E-03	I= 0.74505806E-03	F= 0.	B= 0.						
Coefficients for BODY NO. 2 OF SURFACE NO. 2	A= 0.	E= 0.	I= 0.	F= 0.	B= 0.						
C= 0.	D= 0.	H= 0.	T= 0.	J= 0.09999999E 01							
G= 0.	3.	1.	1.								
	0.	0.	2.750	0.	1.250	2.750	1.250				
Q=11,0,0) FOR BODY NO. 2	OF SURFACE NO. 3										
Coefficients for BODY NO. 2 OF SURFACE NO. 3	A= 0.	E= 0.	I= 0.	F= 0.	B= 0.						
Coefficients for BODY NO. 2 OF SURFACE NO. 3	A= 0.	E= 0.	I= 0.	F= 0.	B= 0.						
C= 0.	D= 0.	H= 0.	T= 0.	J= 0.09999999E 01							
G= 0.	4.	1.	1.								
	6.	4.	1.	1.	-1.000	0.	0.				
Coefficients for BODY NO. 1 OF SURFACE NO. 4	A= 0.	E= 0.	I= -0.16969696E 01	F= 0.	B= 0.						
Coefficients for BODY NO. 1 OF SURFACE NO. 4	A= 0.	E= 0.	I= -0.16969696E 01	F= 0.	B= 0.						
C= 0.49848484E-00 D= 0.	H= 0.	T= 0.	J= 0.09999999E 01								
G= 0.	6.	4.	2.								
	6.	4.	3.	2.	-1.000	0.	0.				
Coefficients for BODY NO. 2 OF SURFACE NO. 4	A= 0.	E= 0.	I= 0.09999999E 01	F= 0.	B= 0.						
Coefficients for BODY NO. 2 OF SURFACE NO. 4	A= 0.	E= 0.	I= 0.09999999E 01	F= 0.	B= 0.						
C= 0.	D= 0.	H= 0.	T= 0.	J= 0.							
G= 5.	6.	4.	1.								
	5.	6.	1.	0.	0.	0.	0.				

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TABLE 6-4 (Continued)

1.000	-1.000	-1.000	0.	0.	0.	-4.000
Q=10.0, FOR BODY NO. 1 OF SURFACE NO. 6						
COEFFICIENTS FOR BODY NO. 1 OF SURFACE NO. 6						
C= -0.2500000E-00	D= 0.	E= 0.	F= 0.2500000E-00	A= 0.2500000E-00	B= 0.	-0.2500000E-00
G= -0.09999999E 01	H= 0.	I= 0.	J= 0.09999999E 01	K= 0.	L= 0.09999999E 01	M= 0.
6	7	1	1	-1.000		
Q=10.0, FOR BODY NO. 2 OF SURFACE NO. 7						
COEFFICIENTS FOR BODY NO. 2 OF SURFACE NO. 7						
C= -0.09999999E 01	D= 0.	E= 0.	F= 0.	A= 0.	B= 0.	-0.09999999E 01
G= 0.	H= 0.	I= 0.	J= 0.	K= 0.	L= 0.	M= 0.
6	8	1	1	-1.000		
COEFFICIENTS FOR BODY NO. 1 OF SURFACE NO. 8						
C= -0.2500000E-00	D= 0.	E= 0.	F= 0.2500000E-00	A= 0.2500000E-00	B= 0.	-0.2500000E-00
G= -0.09999999E 01	H= 0.	I= 0.	J= 0.09999999E 01	K= 0.	L= 0.09999999E 01	M= 0.
6	9	2	2	-1.000		
COEFFICIENTS FOR BODY NO. 2 OF SURFACE NO. 8						
C= -0.09999999E 01	D= 0.	E= 0.	F= 0.	A= 0.	B= 0.	-0.09999999E 01
G= 0.	H= 0.	I= 0.	J= 0.	K= 0.	L= 0.	M= 0.
5	9	1	1	1.000		
0.	0.	4.000	0.	0.	0.	0.
0.	1.000	-3.000				
Q=10.0, FOR BODY NO. 1 OF SURFACE NO. 9						
COEFFICIENTS FOR BODY NO. 1 OF SURFACE NO. 9						
C= -0.1333333E 01	D= -0.	E= 0.	F= -0.	A= -0.	B= -0.	-0.
G= -0.	H= -0.	I= 1.000	J= 1.000	K= -0.3333333E-00	L= 0.09999999E 01	M= 0.
6	10	1	1	9	1	1.000
Q=10.0, FOR BODY NO. 2 OF SURFACE NO. 10						
COEFFICIENTS FOR BODY NO. 1 OF SURFACE NO. 10						
C= -0.1333333E 01	D= -0.	E= -0.	F= -0.	A= -0.	B= -0.	-0.
G= -0.	H= 2	I= 0	J= 1.000	K= -0.3333333E-00	L= 0.09999999E 01	M= 0.
4	10	2	0	1.000	1.000	2.000
0.	0.	0.	0.	-1.000	0.	-2.000
2.000	0.	1.000	0.	1.000	0.	2.000
0.	0.	0.	1.000	1.000	0.	2.000
Q=10.0, FOR BODY NO. 2 OF SURFACE NO. 10						
COEFFICIENTS FOR BODY NO. 1 OF SURFACE NO. 10						
C= -0.1333333E 01	D= -0.	E= -0.	F= -0.	A= -0.	B= -0.	-0.
G= -0.	H= 0	I= 0	J= 1.000	K= -0.3333333E-00	L= 0.09999999E 01	M= 0.
4	10	0	0	0	1.000	2.000
0.	0.	0.	0.	1.000	0.	2.000
0.	0.	0.	1.000	1.000	0.	2.000



TABLE 6-h (Continued)

COEFFICIENTS FOR BODY NO. 7 OF SURFACE NO. 10 A= 0.09999999E 01 B= -0.09999999E 01						
C=	-0.	D=	-0.	F=	0.	
G=	-0.	H=	0.	I=	-0.	
6	11	1	10	1	1.000	

COEFFICIENTS FOR BODY NO. 1 OF SURFACE NO. 11 A= -0. E= -0. F= -0. G= -0.						
C=	-0.13333333E 01	D=	-0.	E=	-0.	
G=	-0.	H=	0.	I=	-0.33333333E-00	J=
6	11	2	10	2	-1.000	

COEFFICIENTS FOR BODY NO. 2 OF SURFACE NO. 11 A= 0.09999999E 01 B= -0.09999999E 01						
C=	-0.	D=	-0.	F=	0.	
G=	-0.	H=	0.	I=	-0.	
7	12	5	0	-0	-0	



TABLE 6-4 (Continued)

-1	1	2	-0	-0
SURFACE NUMBER	1	WAS PUT ON TAPE		
SURFACE NUMBER	2	WAS PUT ON TAPE		
0	3	4	-0	-0
SURFACE NUMBER	3	WAS PUT ON TAPE		
SURFACE NUMBER	4	WAS PUT ON TAPE		
0	5	6	-0	-0
SURFACE NUMBER	5	WAS PUT ON TAPE		
SURFACE NUMBER	6	WAS PUT ON TAPE		
0	7	8	-0	-0
SURFACE NUMBER	7	WAS PUT ON TAPE		
SURFACE NUMBER	8	WAS PUT ON TAPE		
0	9	10	-0	-0
SURFACE NUMBER	9	WAS PUT ON TAPE		
SURFACE NUMBER	10	WAS PUT ON TAPE		
0	11	12	-0	-0
SURFACE NUMBER	11	WAS PUT ON TAPE		
1	12	13	-0	-0
DELS	0.16666666E-00	0.16666666E-00	0.62500000E-01	0.62500000E-01
S= 2	T= 5	A(S,T)=	0.12440328E-00	
S= 2	AREA (S)=	0.12269313E 02		
S= 2	T= 5	F(S,T)=	0.10057412E-01	
1	9	2	-0	-0
DELS	0.93750000E-01	0.10937500E-02	0.16666666E-00	0.16666666E-00
S= 9	T= 2	A(S,T)=	0.13303779E-01	
S= 9	AREA (S)=	0.74552698E 01		
S= 9	T= 2	F(S,T)=	0.17344822E-02	
1	5	9	-0	-0
DELS	0.62500000E-01	0.62500000E-01	0.93750000E-01	0.93750000E-01
S= 5	T= 9	A(S,T)=	0.21098252E-00	
S= 5	AREA (S)=	0.32967237E 01		

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TABLE 6- $\frac{1}{4}$ (Continued)

S= 5	T= 9	$F(S,T)=$	0.63997636E-01
			-0 -0
DELS	0.11705117E-00	0.13333333E-00	0.44194186E-01
S= 3	T= 6	$A(S)F(S,T)=$	0.34657088E-01
S= 3	AREA (S)=	0.63399498E 01	
S= 3	T= 6	$F(S,T)=$	0.54664609E-02
1	3		-0 -0
DELS	0.11705117E-00	0.13333333E-00	0.44194186E-01
S= 3	T= 7	$A(S)F(S,T)=$	0.1510493E-01
S= 3	AREA (S)=	0.63399498E 01	
S= 3	T= 7	$F(S,T)=$	0.30773892E-02
1	3		-0 -0
DELS	0.11705117E-00	0.13333333E-00	0.44194186E-01
S= 3	T= 8	$A(S)F(S,T)=$	0.61239099E-01
S= 3	AREA (S)=	0.63399498E 01	
S= 3	T= 8	$F(S,T)=$	0.86592401E-02
1	4		-0 -0
DELS	0.11705117E-00	0.13333333E-00	0.44194186E-01
S= 4	T= 6	$A(S)F(S,T)=$	0.43677874E-02
S= 4	AREA (S)=	0.63399498E 01	
S= 4	T= 6	$F(S,T)=$	0.68803088E-03
1	4		-0 -0
DELS	0.11705117E-00	0.13333333E-00	0.44194186E-01
S= 4	T= 7	$A(S)F(S,T)=$	0.65088601E-02
S= 4	AREA (S)=	0.63399498E 01	
S= 4	T= 7	$F(S,T)=$	0.95544950E-03
1	4		-0 -0



TABLE 6-4 (Continued)

DELS	0.11705117E-00	0.13333333E-00	0.44194249E-01	0.44194186E-01
S= 4	T= 8	A(S)F(S,T)=	0.63545395E-02	
S= 4	AREA (S)=	0.6339969AE 01		
S= 4	T= 8	F(S,T)=	0.10023011E-02	
1	10		-0	
DELS	0.66291286E-01	0.10937500E-00	0.11785117E-00	0.13333333E-00
S= 10	T= 3	A(S)F(S,T)=	0.81876917E-02	
S= 10	AREA (S)=	0.41190552E 01		
S= 10	T= 3	F(S,T)=	0.21326580E-02	
1	10		-0	
DELS	0.66291286E-01	0.10937500E-00	0.11785117E-00	0.13333333E-00
S= 10	T= 4	A(S)F(S,T)=	0.74165173E-03	
S= 10	AREA (S)=	0.41190552E 01		
S= 10	T= 4	F(S,T)=	0.19001441E-03	
1	6		-0	
DELS	0.44194106E-01	0.44194186E-01	0.66291286E-01	0.10937500E-00
S= 5	T= 10	A(S)F(S,T)=	0.12823909E-00	
S= 6	AREA (S)=	0.95819621E 00		
S= 6	T= 10	F(S,T)=	0.13383469E-00	
1	7		-0	
DELS	0.44194249E-01	0.44194186E-01	0.66291286E-01	0.10937500E-00
S= 7	T= 10	A(S)F(S,T)=	0.28205830E-01	
S= 7	AREA (S)=	0.11303678E 01		
S= 7	T= 10	F(S,T)=	0.26952709E-01	
1	8		-0	
DELS	0.44194249E-01	0.44194186E-01	0.66291286E-01	0.10937500E-00

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TABLE 6-4 (Continued)

S= 8	R	T= 10	A(S)F(S,T)=	0.37123086E-01
S= 8		AREA (S)=	0.11303678E 01	
S= 8	1	T= 10 6	F(S,T)=	0.32841597E-01 11 -0
DELS		0.441194186E-01	0.441194186E-01	0.66291286E-01 0.10937500E-00
S= 6	7	T= 11	A(S)F(S,T)=	0.84201583E-02
S= 6		AREA (S)=	0.95019621E 00	
S= 6	1	T= 11 7	F(S,T)=	0.87875094E-02 11 -0
DELS		0.441194224E-01	0.441194186E-01	0.66291286E-01 0.10937500E-00
S= 7	7	T= 11	A(S)F(S,T)=	0.75947961E-02
S= 7		AREA (S)=	0.11303678E 01	
S= 7	1	T= 11 8	F(S,T)=	0.67108713E-02 11 -0
DELS		0.441194249E-01	0.441194186E-01	0.66291286E-01 0.10937500E-00
S= 8	8	T= 11	A(S)F(S,T)=	0.36494599E-02
S= 8		AREA (S)=	0.11303678E 01	
S= 8	3	T= 11 1	F(S,T)=	0.32285595E-02 11 -0 -0
S= 1	3		AREA (S)=	0.23840413E 03 11 -0 -0 -0
S= 11	4		AREA (S)=	0.4119552E 01 -0 -0 -0 -0



$x = 0, y = \pm .75$ are "vertical" points on the cylinder with x -dependent. The values of the AF product are .0133 with one dependent variable or .0191 for the partitioned surfaces. From the sphere to the cylinder, the AF product is .211 with x -dependent and .213 partitioned. This relatively small difference in the AF product is due to the fact that the area along the curves of vertical points has relatively little effect. Note that the lines $x = 0, y = \pm .75$ on the cylinder cannot be seen from any point on the sphere; thus, the areas along these lines will not affect the AF products.

The shape factor for the partitioned surfaces can be computed by dividing the total AF product by the area computed as the sum of the subareas. This gives the comparative values of .0101 and .0104 for the shape factor from the cone to sphere. The largest difference is in the case of the shape factor from the cylinder to cone which changes from .00178 to .00231 due to the partitioning. Although there is relatively little difference in the AF products between the sphere and cylinder, the difference in the area of the sphere produces a change from .0640 to .0662 for the shape factor.

From this it can be seen that partitioning gives more accurate values for the areas of the cone, sphere and cylinder. In general, with one dependent variable, if the surface has a curve of vertical points, the accuracy of the computed area is strictly by chance with no way to eliminate the possibility of a blow-up as noted on the area of $S(1)$. On the other hand, if a surface is partitioned so that there is no curve of vertical points on any subsurface, the maximum percentage of error is proportional to $K/(GS)^2$ where K is a constant for any given surface shape and partitioning method assuming that the same value for GS is used for each subarea. Since the area generally affects the value of the AF product, the accuracy of the area is usually reflected in the AF product, the shape factor, or both.

A summary of the computed AF products, areas and shape factors is given in Table 6-5.



TABLE 6-5
SUMMARY OF EXAMPLE PROBLEM #2

From Surface s	To Surface t	AF Product A(s)F(s,t)	Area of Surface s, A(s)		F(s,t)
			Computed	True	
Cone S(1)			238.4	12.94	
Cone S(2)	Sphere S(5)	.1244	12.37	12.94	.01006
Cone S(3) + S(4)	Sphere S(6) + S(7) + S(8)	.1321	12.68	12.94	.01042
Sphere S(5)	Cylinder S(9)	.2110	3.297	3.142	.064001
Sphere S(6) + S(7) + S(8)	Cylinder S(10) + S(11)	.2132	3.219	3.142	.06624
Cylinder S(9)	Cone S(2)	.01330	7.455	8.242	.001784
Cylinder S(10) + S(11)	Cone S(3) + S(4)	.01906	8.240	8.242	.002313

EXAMPLE PROBLEM #3

The third example problem deals with a sphere inside a cube. While this is not a strictly practical problem, it is similar to any problem involving an enclosed component or storage tank. The particular geometry, however, was chosen to test the accuracy of the program. For this reason a geometry was chosen for which the shape factors as well as the surface areas can be computed. With the surface areas and shape factors the AF products can also be obtained. Thus, every value computed by the program can be checked for accuracy.

The geometry is illustrated in Figure 6-15. The sphere with radius 2 and the cube which is 4.8 units on the edge are centered at the origin. The top and bottom of the cube are parallel to the xy plane, while the front (called surface S(1)) lies in the vertical plane through (3,0,0) and (0,4,0). Shape factors are computed from the front to the top, to one side, and to the back of the cube, with and without interference by the sphere, as well as shape factors from the sphere to the front and top of the cube. Due to symmetry,

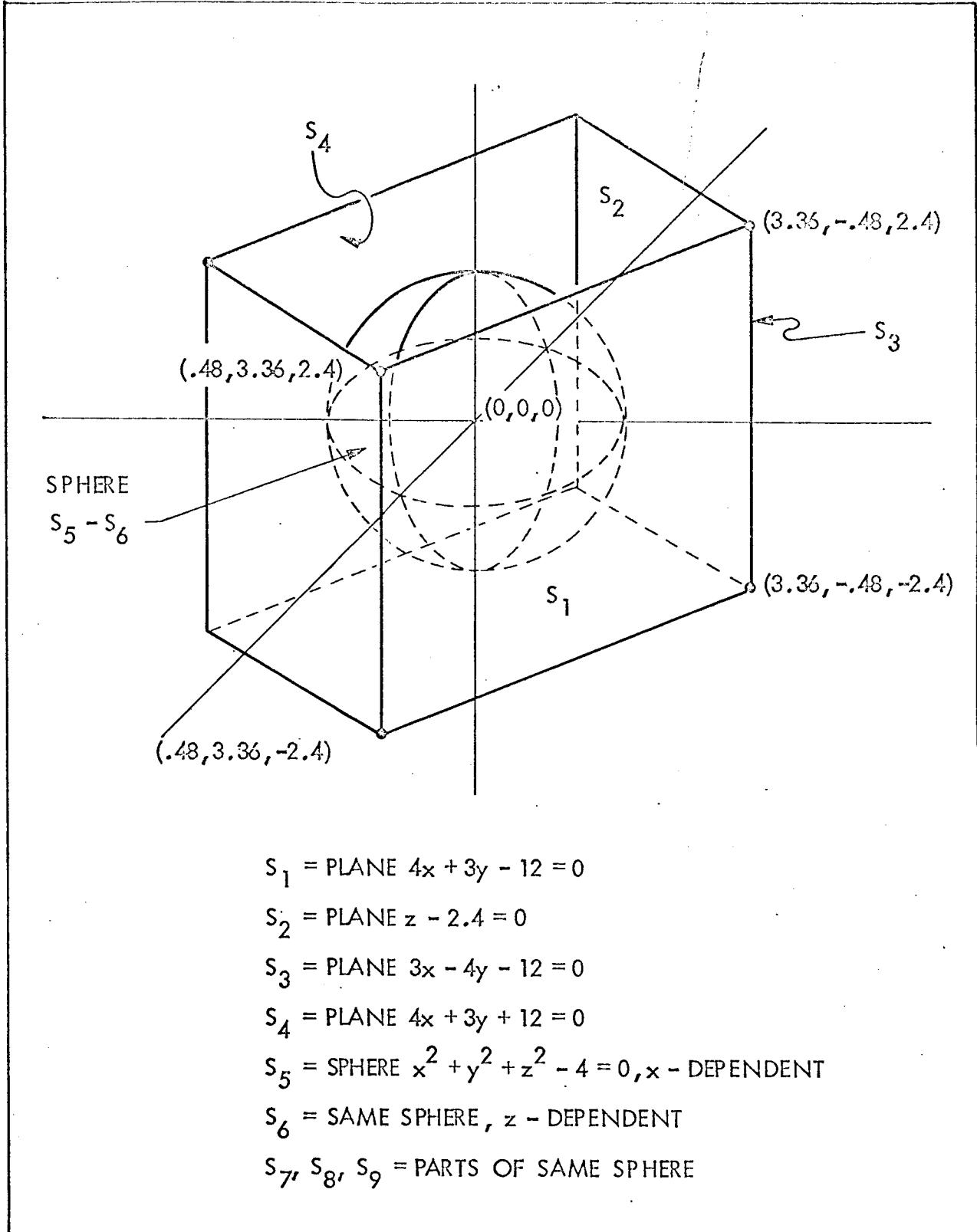


Figure 6-15. Example Problem #3

the remaining shape factors duplicate these. Due to the orientation of the top of the cube, its area is not computed exactly. Thus, the shape factors involving the top are also computed for comparison with the values obtained using the vertical surfaces.

Surface Input

Cube - Four of the six surfaces of the cube are specified as surfaces S(1) to S(4). S(1) (the front) lies on the plane $4x + 3y - 12 = 0$. S(2) (the top) lies on the plane $5z - 12 = 0$. S(3) is the far side of the cube and it is on the plane $3x - 4y - 12 = 0$.

S(4) (the back) is on $4x + 3y + 12 = 0$. The near side on $3x - 4y + 12 = 0$ and the bottom on $5z + 12 = 0$ are not used due to the symmetry of the problem.

S(1) is defined by input Flag 0 for a plane and the three surface points (.448, 3.36, 2.4) (.48, 3.36, -2.4), and (3.36, -.48, -2.4). The dependent variable on S(1) is x and three boundaries are used. The reference point (0,0,0) is at the center of the cube. Therefore, the inside is specified by the side index 1.0. The limits on the independent coordinates are $-.48 \leq y \leq 3.36$ and $-2.4 \leq z \leq 2.4$ while GS is arbitrarily taken as 15.

S(2) is defined by input Flag 0 for a plane and the three surface points (.48, 3.36, 2.4), (3.36, -.48, 2.4), and (-3.36, .48, 2.4). The dependent variable is z and three boundaries are used. The reference point (0,0,0) is below the plane. Thus, the side index is 1.0 for the inside of the cube. The limits on the independent coordinates are $-3.36 \leq x \leq 3.36$ and $-3.36 \leq y \leq 3.36$ with GS again equal to 15.

S(3) is defined by input Flag -1 and the coefficients 0, 0, 0, 0, 0, 0, 3, -4, 0, and -12. The dependent variable is y and there are two boundaries. The surface reference point is (0,0,0) so the inside of the cube is defined by a side index of 1.0. The limits on the independent coordinates are $-.48 \leq x \leq 3.36$ and $-2.4 \leq z \leq 2.4$ with GS equal to 15.

S(4) is defined by input Flag -1 and the coefficients 0, 0, 0, 0, 0, 0, 4, 3, 0, and 12. The dependent variable is y and S(4) has three boundaries.

The reference point is (0,0,0) and the side index is 1.0. The limits on the independent coordinates are $-3.36 \leq x \leq -.48$ and $-2.4 \leq z \leq 2.4$ and GS is 15.

Sphere - The remaining surfaces give all or part of the sphere $x^2 + y^2 - 4 = 0$. S(5) and S(6) show the entire sphere with different dependent variables. Surfaces S(7) and S(8), S(9) and S(10) give parts of the sphere.

S(5) is defined by input Flag 1 and the coordinates (0,0,0) of the center plus the radius 2. The dependent variable is x and there are no boundaries. The reference point is (0,0,0) which is the center of the sphere. Thus, the outside of the sphere is defined by a side index of -1.0. The limits on the independent coordinates are $-2 \leq y \leq 2$ and $-2 \leq z \leq 2$, with GS equal to 15.

S(6) is defined by input Flag 2 plus the four surface points (0,0,2), (2,0,0), (0,2,0), and (0,0,-2). On S(6) z is dependent with no boundaries. The surface reference point is (0,0,0) and the side index is -1.0. The limits on the independent coordinates are $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$, while GS is 15.

S(7) is defined by input Flag 8 and the number 6 of S(6) whose equation is to be used for S(7). The dependent variable is x and one boundary is required. The surface reference point is (0,0,0) and the side index -1.0. The limits on the independent coordinates are $-1.414214 \leq y \leq 1.414214$ and $-1.414214 \leq z \leq 1.414214$, while GS is 15.

S(8) is defined by input Flag 8 and the surface number 6. The dependent variable is y and there are two boundaries. The origin is the reference point and -1.0 the side index. The limits on the independent coordinates are $-1.414214 \leq x \leq 1.414214$ and $-1.414214 \leq z \leq 1.414214$, with GS equal to 15.

S(9) is defined by input Flag 8 and the surface number 6. The dependent variable is z and there are two boundaries. The origin is the reference point and -1.0 the side index. The limits on the independent coordinates are $-1.414214 \leq x \leq 1.414214$ and $-1.414214 \leq y \leq 1.414214$, while GS is 15.

S(10) is defined by input Flag 8 and the surface number 6. The dependent variable is z and three boundaries are used. The side index is -1.0 and the reference point is the origin. The limits on the independent coordinates are $-1.414214 \leq x \leq 1.414214$ and $-1.414214 \leq y \leq 1.414214$, with GS equal to 15.

Boundary Input

Boundaries of the Sides of the Cube - The six planes containing the faces of the cube are used as boundaries either simply or combined into double-plane cylinders.

Boundary $B(1,1)$, which consists of the parallel planes $5z - 12 = 0$ and $5z + 12 = 0$ of the top and bottom of the cube combined as the cylinder $25z^2 - 144 = 0$, is defined by input Flag 5 plus the coefficients 0, 0, 25, 0, 0, 0, 0, 0, and -144 of the equation. The reference point (0,0,0) for this surface is between the two planes. Therefore, the side index 1.0 is used.

Boundaries $B(3,1)$ and $B(4,1)$, which duplicate $B(1,1)$, are given by input Flag 6, the surface number 1, and the boundary number 1. The side indices remain 1.0. $B(1,2)$ duplicates $S(3)$ and is defined by input Flag -1 plus the surface number 3. The reference point is the origin and the side index is 1.0. $B(4,2)$ duplicates $B(1,2)$ and is defined by input Flag 6 plus the surface number 1 and the boundary number 2.

$B(1,3)$ is the near side of the cube ($3x - 4y + 12 = 0$), which is defined by input Flag 0 plus the two boundary points (.48, 3.36, 2.4) and (.48, 3.36, -2.4). The side index and reference point are 1.0 and the origin, respectively.

$B(4,3)$, which duplicates $B(1,3)$, is defined by input Flag 6 plus the surface number 1 and the boundary number 3. The side index remains 1.0.

$B(2,1)$, consisting of the two perpendicular planes $S(1)$ and $S(3)$ combined into the cylinder $12x^2 - 12y^2 - 7xy - 84x + 12y + 144 = 0$, is defined by input Flag 5 plus the coefficients A through J of the equation. The surface reference point (0,0,0) and the required portion of $S(2)$ are in the same angle of these planes. Thus, the side index is 1.0.

$B(2,2)$ consisting of the back $S(4)$, is defined by input Flag -1 and the surface number 4. The side index is 1.0 and the origin is the reference point.

$B(2,3)$, consisting of the rear side ($3x - 4y + 12 = 0$) of the cube, is defined by input Flag 0 plus the boundary points (-3.36, .48, 2.4) and (.48, 3.36, 2.4).



The reference point is $(0,0,0)$ and the side index is 1.0. Note that this does not give a closed system of boundaries for $S(2)$ since there is an unbounded area in the opposite angle of $B(2,1)$ which is not closed by either $B(2,2)$ or $B(2,3)$. Since exact limits are specified for the independent variables on $S(2)$ only one row of grid points crosses the unbounded area. Depending on the orientation of the boundary planes one or more of these grid points will be in the interior of the region. In this case, the cylinder of intersecting planes $B(2,1)$ produces one extraneous grid point. This adds $.200704 = [(6.72)^2/(15)^2] \cos(0^\circ)$ units to the area of $S(2)$. Thus the computed area given in Table 6-8 should be 22.68 or 1.6 percent low.

$B(3,2)$, consisting of the front, $S(1)$, and the back, $S(4)$, of the cube combined as the cylinder $16x^2 + 9y^2 + 24xy - 144 = 0$ is defined by input Flag 4, the five boundary points $(.48, 3.36, 2.4)$, $(3,0,0)$, $(3.36, -.48, -2.4)$, $(-3.36, .48, 2.4)$, and $(0,-4,0)$ plus the direction numbers $(0: 0: 1)$ of an element of the cylinder. Since the surface reference point $(0,0,0)$ and the required area on $S(3)$ lie between these planes, the side index is 1.0.

Boundaries of the Sphere - $S(5)$ and $S(6)$ require no boundaries, but the sections $S(7)$, $S(8)$, $S(9)$, and $S(10)$ of the sphere each require boundaries. $S(7)$, $S(8)$, and $S(9)$ partition the sphere in the same way that the sphere in example problem #2 was partitioned, while $S(10)$ gives one half of $S(9)$ by using the two planes of $B(9,2)$ individually as separate boundaries.

$B(7,1)$, consisting of the cone $x^2 - y^2 - z^2 = 0$, is defined by input Flag 5 plus the ten coefficients A through J of the equation of the cone. The surface reference point $(1,0,0)$ and the side index 1.0 define the two caps of the partitioned sphere, which are centered on the x-axis, as shown in Figure 6-16.

$B(8,1)$, $B(9,1)$ and $B(10,1)$ all duplicate boundary surface $B(7,1)$. Thus they are defined by input Flag 6, the surface number 7 and the boundary surface number 1. Since these surfaces, $S(8)$, $S(9)$, and $S(10)$, are on the outside of the cone, the side index for each of these boundary surfaces is -1.0.

$B(8,2)$, consisting of $y^2 - z^2 = 0$ which combines the two planes $y - x = 0$ and $y + z = 0$ into a cylinder, is defined by input Flag 4 plus the five boundary points $(0,0,0)$, $(0,1,1)$, $(0,-1,-1)$, $(0,1,-1)$, and $(0,-1,1)$,



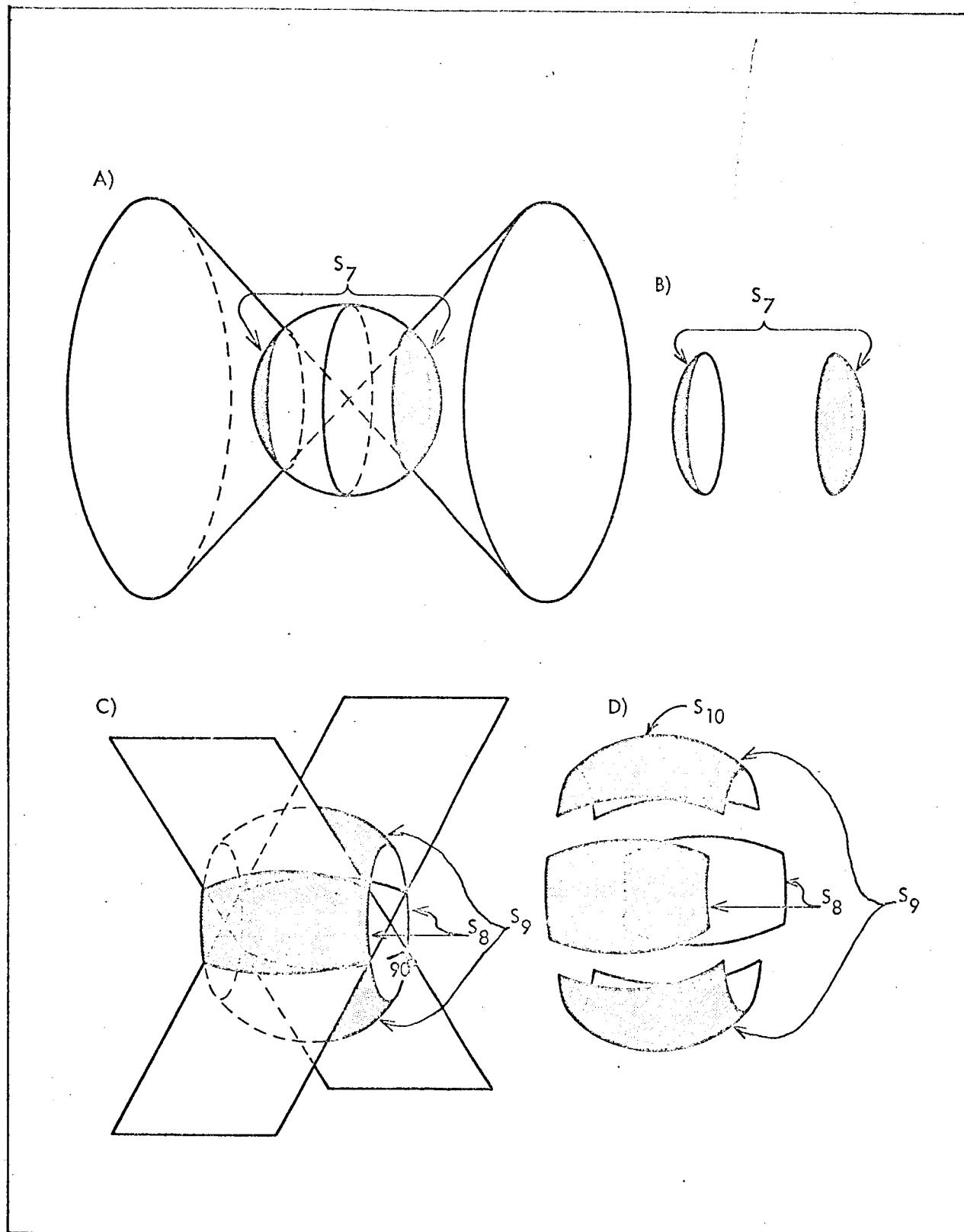


Figure 6-16. Example Problem #3 - Partitioned Sphere

together with the direction numbers (1: 0: 0) of an element of the cylinder. The surface reference point is (0,1,0) and the side index is 1.0.

$B(9,2)$, which duplicates $B(8,2)$, is defined by input Flag 6 plus the surface number 8 and the boundary number 2. Since $S(8)$ lies in the angles of $y^2 - z^2 = 0$ containing the xy -plane, while $S(9)$ lies in the angles containing the xz -plane, the side index is -1.0 for $B(9,1)$.

$B(10,2)$, consisting of $y - z = 0$, is defined by input Flag 1 plus the three boundary points (2,0,0), (-2,0,0), and (1.414214, 1, 1). The reference point is (0,1,0) and the side index -1.0.

$B(10,3)$, consisting of $y + z = 0$ is defined by input Flag 1 plus the three boundary points (2,0,0), (-2,0,0), and (1.414214, 1, -1). The reference point is (0,1,0) and the side index is 1.0. Note that while the union of the $B(10,2)$ and $B(10,3)$ produces the same boundary surface as $B(9,2)$, $S(9)$ and $S(10)$ are not the same. The two boundary surfaces, $B(10,2)$ and $B(10,3)$, acting together give one half of $S(9)$, namely the half that intersects the positive z -axis, while rejecting the part which intersects the negative z axis.

Computation Control

Grid points are required for all ten surfaces. The surface-to-surface shape factors $F(1,2)$, $F(1,3)$, $F(1,4)$, and $F(2,1)$ are computed as if the sphere were not inside the cube. The shape factors $F(1,2)$, $F(1,3)$, and $F(1,4)$ are then computed with interference by the sphere $S(5)$. The shape factors $F(1,5)$, $F(5,1)$, $F(6,1)$, $F(7,1)$, $F(8,1)$, $F(9,1)$, $F(5,2)$, and $F(6,2)$ are computed between all or part of the sphere and the front and top of the cube. To indicate the relative importance of the areas at the intersection of the coordinate axes and the sphere, as well as the area at the point on the sphere closest to $S(1)$ in the computation of the shape factor from the sphere to the front plane of the cube, eight point-to-surface shape factors are requested by supplying four sets of independent coordinates. Since each of the three pairs of points (0,0, ± 2), (0, ± 2 ,0) and (± 2 ,0,0) requires a different dependent variable, three entries are required. The independent coordinates 1.2 and 0 of the closest point (1.6, 1.2, 0) and its companion point (-1.6, 1.2, 0) are added to the independent coordinates 0 and 0 of (± 2 ,0,0) in the third entry. The computation of the area $A(1)$ of $S(10)$ completes the computation.

Discussion of Problem

The input sheets and the output for this problem are shown in Tables 6-6 and 6-7. In the closed symmetric geometry of this problem all the shape factors can be determined accurately from outside information and reasoning. The only outside information required is the well known result that the shape factor from a square to a parallel square is .2 if the squares are exactly opposite each other and the distance between them is equal to the length of a side. These conditions are exactly met by the front and back, top and bottom, etc., of the cube.

From this it can be reasoned that the shape factor from any surface of the cube to any other surface of the cube must be .2. Since the cube is closed, the sum of the shape factors from one side to each of the other five sides must be 1, but the factor to the opposite side is known to be .2. Thus the sum of the shape factors to the four adjacent sides is $1 - .2 = .8$. By symmetry these four shape factors must be equal and the shape factor from one side of the cube to an adjacent side is $.8/4 = .2$. Thus the shape factor from any side of the cube to any other side of the cube is .2.

Consider the case where the sphere is in the center of the cube. Since the sphere is completely enclosed, the sum of these shape factors must be 1. Again, by symmetry, the shape factors from the sphere to each of the six sides of the cube must be equal; therefore, from the sphere to any one side the shape factor is $1/6$. Using the fact that $A(1)F(1,2)$ equals $A(2)F(2,1)$, the shape factor from a side to the sphere can also be computed. The area of sphere of radius 2 is 50.26, while the area of each of the sides is 23.04. Thus, the AF product is $50.26 \times 1/6 = 8.377 \times F(2,1)$. From the shape factor $F(2,1)$ from plane to sphere can be obtained as $8.377/23.04 = .3636$.

The shape factors from side to side with interference by the sphere cannot be computed, but it can be seen that the factors to the other four sides should be equal and considerably larger than the factor to the opposite side, while the sum of these five factors should equal $1 - .3636 = .6364$.

Table 6-8 shows the computed and actual values for the AF products, the areas and shape factors computed for this problem. The top entry in each case is the computed value while the bottom is the true one. The identification sphere -X, sphere -Z and sphere -P, respectively, are used in this table for

TABLE 6-6 EXAMPLE PROBLEM #3 INPUT

SEQ.	10
001	77 80 1
002	5 15 20
003	0 19 25
004	-1/2 26
005	5 15
006	-3 26
007	3 26
008	3 26
009	-1
010	-4/3 26
011	0 1
012	-1 2
013	-1
014	-3 26
015	0 1
016	3 26
017	1
018	-1/2 26
019	0 1
020	5 15
021	-1/2 26
022	0 1
023	1
024	8 15
025	1/4 26 1/4 26 1/4 26

TABLE 6-6 (continued)

SEQ.	GENERAL PURPOSE DATA SHEET										LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION										PAGE OF		
	PREPARED	NAME	DATE	TITLE										NO.	EWA	GROUP	JOB NO.						
				NO.																			
ID				15	10	5	0	25	20	15	10	5	0	35	30	25	20	15	10	5	0	65	70
77 801				8	8	8	8	2	2	2	2	2	2	6	6	6	6	6	6	6	6	65	70
026				-1.	4	2	1	4	2	1	4	2	1	4	-1.	4	2	1	4	1.	4	2	1
027-1.				8	9	9	9	3	3	3	3	3	3	6	6	6	6	6	6	6	6	6	65
028				-1.	4	1	4	2	1	4	1	4	2	1	4	-1.	4	1	4	2	1	4	1
029-1.				8	9	9	9	3	3	3	3	3	3	6	6	6	6	6	6	6	6	6	65
030				5	5	5	5	2	2	2	2	2	2	6	6	6	6	6	6	6	6	6	65
031-1.				-1.	4	1	4	2	1	4	1	4	2	1	-1.	4	1	4	2	1	4	1	4
032				9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
033				5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
034	0.			0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
035	0.			5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
036				-1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3
037				0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
038	48			3.36	2.4	2.4	2.4	4.2	4.2	4.2	4.2	4.2	4.2	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36
039				5	2	2	2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
24012.				-12.	0.	0.	0.	-7.	-7.	-7.	-7.	-7.	-7.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
041	12.			0.	0.	0.	0.	144.	144.	144.	144.	144.	144.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
042				-1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
043				0	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
044	-3.	36		.48	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
045				6	3	3	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
046				4	3	3	3	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0
047	48			3.36	2.4	2.4	2.4	3	3	3	3	3	3	2.36	2.36	2.36	2.36	2.36	2.36	2.36	2.36	2.36	2.36
048	-48			-2.	3.6	3.6	3.6	4.8	4.8	4.8	4.8	4.8	4.8	3	3	3	3	3	3	3	3	3	3
049	0.			0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
050				0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.



TABLE 6-6 (Continued)

SEQ.	GENERAL PURPOSE DATA SHEET										LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION										PAGE OF JOB NO.
	PREPARED	NAME			DATE			TITLE			WD			EWA			GROUP				
CHECKED																					
ID	77	80	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	72	73	76	
051																					
052																					
053																					
054																					
055	0.																				
056																					
057																					
058	0.																				
059	-1.																				
060																					
061																					
062																					
063																					
064																					
065	2.																				
066																					
067																					
068	2.																				
069	1.																				
070																					
071																					
072																					
073																					
074																					
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TABLE 6-6 (Continued)

SEQ.	GENERAL PURPOSE DATA SHEET										LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION										PAGE OF JOB NO.				
	PREPARED		NAME		DATE		TITLE		#0					EWA					GROUP						
	CHECKED																								
SEQ.	10	5	10	20	25	30	35	40	45	50	55	60	65	70	72	73	76	78	79	80	81	82	83	84	
076	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
077	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
078	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
079	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
080	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
081	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
082	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
083	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
084	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
085	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
086	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
087	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
088	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
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090	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
091	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
092	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
093	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
094	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
095	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
096	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
097	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
098	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	
099	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	



TABLE 6-6 (Continued)

SEQ.	GENERAL PURPOSE DATA SHEET			LOCKHEED-CALIFORNIA COMPANY A DIVISION OF LOCKHEED AIRCRAFT CORPORATION												PAGE OF	
	PREPARED	NAME	DATE	TITLE			V.O.			FWA			GROUP			JOB NO.	
CHECKED																	
ID				1	2	3	4	5	6	7	8	9	10	11	12	13	14
77-80-1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	72	73	75
101	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
102	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
103	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
104	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	72	73	75
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	72	73	75
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23



TABLE 6-7. OUTPUT FOR EXAMPLE #3



TABLE 6-7 (Continued)

	8	7	1	1	1	6	
	-1.000	-1.414	1.414	-1.414	1.414	15.000	
COEFFICIENTS FOR SURFACE NO. 7							
C= -0.2500000E-00	D= -0.	A= -0.2500000E-00	B= -0.2500000E-00				
F= -0.	H= 0.	I= -0.	J= -0.				
5= -7.	6= 2.						
-1.000	8	-1.414	1.414	-1.414	1.414	15.000	
COEFFICIENTS FOR SURFACE NO. 8							
C= -0.2500000E-00	D= -0.	A= -0.2500000E-00	B= -0.2500000E-00				
G= -7.	H= 0.	I= -0.	J= -0.				
-1.000	R	-1.414	1.414	-1.414	1.414	15.000	
COEFFICIENTS FOR SURFACE NO. 9							
C= -0.2500000E-00	D= -0.	A= -0.2500000E-00	B= -0.2500000E-00				
G= -3.	H= 0.	I= -0.	J= -0.				
-1.000	8	10	-1.414	1.414	-1.414	1.414	15.000
COEFFICIENTS FOR SURFACE NO. 10							
C= -0.2500000E-00	D= -0.	A= -0.2500000E-00	B= -0.2500000E-00				
G= -0.	H= 0.	I= -0.	J= -0.				
9	11	-0	-0	-0	-0	-0	

*Due to the format used only three decimal places are printed.



TABLE 6-7 (Continued)

S	1	1	0	0	1.000			
C.	0.	25.000	0.	0.	0.			
G.	0.	-144.000						
Q=10.0,0) FOR BDRY NO. 1 OF SURFACE NO. 1								
COEFFICIENTS FOR BDRY NO.	1	OF SURFACE NO.	1	A=	-0.			
C= -0.17361110E-00	D=	-0.	E=	-0.	F=	-0.		
G= -0.	H=	-0.	I=	-0.	J=	0.09999999E 01		
-1	1	2	0	1.000				
COEFFICIENTS FOR BDRY NO.								
2 OF SURFACE NO. 1 A= -0.								
C= -0.	D=	-0.	E=	-0.	F=	-0.		
G= -0.25000000E-00	H=	0.33333333E-00	I=	-0.	J=	0.09999999E 01		
-1	1	3	0	1.000				
n.480	3.360	2.400	0.480	3.360	-2.400			
Q=10.0,0) FOR BDRY NO. 3 OF SURFACE NO. 1								
COEFFICIENTS FOR BDRY NO.	3	OF SURFACE NO.	1	A=	-0.			
C= 0.	D=	0.	E=	0.	F=	0.		
G= 0.25000001E-00	H=	-0.33333334E-00	I=	-0.	J=	0.09999999E 01		
-1	2	1	0	1.000				
12.000	-12.000	0.	-7.000	0.	0.	-94.000		
12.000	0.	144.000						
Q=10.0,0) FOR BDRY NO. 1 OF SURFACE NO. 2								
COEFFICIENTS FOR BDRY NO.	1	OF SURFACE NO.	2	A=	0.03333333E-01	B=	-0.83333333E-01	
C= 0.	D=	-0.46111101E-01	E=	0.	F=	0.		
G= -0.53773135E-00	H=	0.83333333E-01	I=	0.	J=	0.09999999E 01		
-1	2	2	0	1.000				
COEFFICIENTS FOR BDRY NO.								
2 OF SURFACE NO. 2 A= 0.								
C= 0.	D=	0.	E=	0.	F=	0.		
G= 0.33333333E-00	H=	0.25000000E-00	I=	0.	J=	0.09999999E 01		
-1	2	3	0	1.000				
-3.360	0.480	2.400	0.480	3.360	2.400			
Q=10.0,0) FOR BDRY NO. 3 OF SURFACE NO. 2								
COEFFICIENTS FOR BDRY NO.	3	OF SURFACE NO.	2	A=	0.	B=	0.	
C= 0.	D=	-0.33333333E-00	E=	0.	F=	0.		
G= 0.24999999E-00	H=	-0.117587E-07	I=	1.000	J=	0.09999999E 01		
-1	3	1	1					
COEFFICIENTS FOR BDRY NO.								
1 OF SURFACE NO. 3 A= -0.								
C= 0.	D=	-0.	E=	-0.	F=	-0.		
G= -0.17361110E-00	H=	-0.	I=	-0.	J=	0.09999999E 01		
-1	4	3	2	0	1.000			
0.480	3.360	2.400	3.000	0.	2.400	3.360		
-n.480	-2.400	-3.360	0.480	2.400	1.000	-4.000		



TABLE 6-7 (Continued)

$Q = (0, 0, 0)$ FOR BDRY NO. 2 OF SURFACE NO. 3	$2.37252903E-07$	-0.	$0.37252903E-07$	-0.44702484E-07	-0.62500002E-01	$-0.74505806E-07$
COEFFICIENTS FOR BDRY NO. 2 OF SURFACE NO. 3	$A = 0.$	$B = -0.11111111E-00$	$C = -0.$	$D = -0.16666667E-00$	$E = -0.$	$F = -0.$
$C = -0.16666667E-00$	$D = 0.10184429E-07$	$E = 1.$	$F = 1.$	$G = 1.000$	$H = 0.09999999E 01$	$I = 0.09999999E 01$
$G = 0.13379239E-07$	$H = 0.10184429E-07$	$I = 1.$	$J = 1.$	$K = 1.$	$L = 1.$	$M = 1.$
$S = 6$	$T = 4$	$U = 1$	$V = 1$	$W = 1$	$X = 1$	$Y = 1$
COEFFICIENTS FOR BDRY NO. 1 OF SURFACE NO. 4	$A = -0.$	$B = -0.$	$C = -0.$	$D = -0.$	$E = -0.$	$F = -0.$
$C = -0.17361110E-00$	$D = -0.$	$E = -0.$	$F = -0.$	$G = -0.$	$H = -0.$	$I = -0.$
$G = -0.$	$H = 2.$	$I = 1.$	$J = 2.$	$K = 1.$	$L = 1.$	$M = 1.$
$S = 6$	$T = 4$	$U = 2$	$V = 1$	$W = 1$	$X = 1$	$Y = 1$
COEFFICIENTS FOR BDRY NO. 2 OF SURFACE NO. 4	$A = -0.$	$B = -0.$	$C = -0.$	$D = -0.$	$E = -0.$	$F = -0.$
$C = -0.$	$D = 0.33333333E-00$	$E = -0.$	$F = -0.$	$G = -0.$	$H = -0.$	$I = -0.$
$G = -0.25000000E-00$	$H = 3.$	$I = 3.$	$J = 3.$	$K = 3.$	$L = 3.$	$M = 3.$
$S = 6$	$T = 4$	$U = 3$	$V = 3$	$W = 3$	$X = 3$	$Y = 3$
COEFFICIENTS FOR BDRY NO. 3 OF SURFACE NO. 4	$A = 0.$	$B = 0.$	$C = 0.$	$D = 0.$	$E = 0.$	$F = 0.$
$C = 0.$	$D = 0.$	$E = 0.$	$F = 0.$	$G = 0.$	$H = 0.$	$I = 0.$
$G = 0.25000001E-00$	$H = 0.$	$I = 0.$	$J = 0.$	$K = 0.$	$L = 0.$	$M = 0.$
$S = 5$	$T = 7$	$U = 1$	$V = 1$	$W = 1$	$X = 1$	$Y = 1$
$A = 1.000$	$B = -1.000$	$C = -1.000$	$D = 0.$	$E = 0.$	$F = 0.$	$G = 0.$
$H = 0.$	$I = 0.$	$J = 0.$	$K = 0.$	$L = 0.$	$M = 0.$	$N = 0.$
$Q = (1, 0, 0)$ FOR BDRY NO. 1 OF SURFACE NO. 7	$A = 0.$	$B = 0.$	$C = 0.$	$D = 0.$	$E = 0.$	$F = 0.$
COEFFICIENTS FOR BDRY NO. 1 OF SURFACE NO. 7	$A = 0.$	$B = 0.$	$C = 0.$	$D = 0.$	$E = 0.$	$F = 0.$
$C = -0.09999999E 01$	$D = 0.$	$E = 0.$	$F = 0.$	$G = 0.$	$H = 0.$	$I = 0.$
$G = 0.$	$H = 0.$	$I = 0.$	$J = 0.$	$K = 0.$	$L = 0.$	$M = 0.$
$S = 6$	$T = 8$	$U = 1$	$V = 7$	$W = 1$	$X = 1$	$Y = 1$
COEFFICIENTS FOR BDRY NO. 1 OF SURFACE NO. 8	$A = 0.$	$B = 0.$	$C = 0.$	$D = 0.$	$E = 0.$	$F = 0.$
$C = -0.09999999E 01$	$D = 0.$	$E = 0.$	$F = 0.$	$G = 0.$	$H = 0.$	$I = 0.$
$G = 0.$	$H = 0.$	$I = 0.$	$J = 0.$	$K = 0.$	$L = 0.$	$M = 0.$
$S = 6$	$T = 9$	$U = 1$	$V = 7$	$W = 1$	$X = 1$	$Y = 1$
COEFFICIENTS FOR BDRY NO. 2 OF SURFACE NO. 8	$A = -0.$	$B = -0.$	$C = -0.$	$D = -0.$	$E = -0.$	$F = -0.$
$C = -0.09999999E 01$	$D = 0.$	$E = 0.$	$F = 0.$	$G = 0.$	$H = 0.$	$I = 0.$
$G = 0.$	$H = 0.$	$I = 0.$	$J = 0.$	$K = 0.$	$L = 0.$	$M = 0.$
$S = 6$	$T = 9$	$U = 1$	$V = 7$	$W = 1$	$X = 1$	$Y = 1$
COEFFICIENTS FOR BDRY NO. 1 OF SURFACE NO. 9	$A = 0.$	$B = 0.$	$C = 0.$	$D = 0.$	$E = 0.$	$F = 0.$
$C = -0.09999999E 01$	$D = 0.$	$E = 0.$	$F = 0.$	$G = 0.$	$H = 0.$	$I = 0.$
$G = 0.$	$H = 0.$	$I = 0.$	$J = 0.$	$K = 0.$	$L = 0.$	$M = 0.$
$S = 6$	$T = 9$	$U = 2$	$V = 8$	$W = 2$	$X = 1.$	$Y = 1.$
COEFFICIENTS FOR BDRY NO. 2 OF SURFACE NO. 9	$A = -0.$	$B = -0.$	$C = -0.$	$D = -0.$	$E = -0.$	$F = -0.$
$C = -0.09999999E 01$	$D = 0.$	$E = 0.$	$F = 0.$	$G = 0.$	$H = 0.$	$I = 0.$
$G = 0.$	$H = 0.$	$I = 0.$	$J = 0.$	$K = 0.$	$L = 0.$	$M = 0.$
$S = 6$	$T = 9$	$U = 2$	$V = 8$	$W = 2$	$X = 1.$	$Y = 1.$

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TABLE 6-7 (Continued)

6 19 1 7 1 -1.000

COEFFICIENTS FOR BODY NO. 1 OF SURFACE NO. 10

C=	-0.0999999E 01	D=	0.	E=	0.	A=	0.099999999E 01	B=	-0.099999999E 01
G=	0.	H=	0.	I=	0.	F=	0.	J=	0.
1	10	2	0.	0.	0.				
2.000	0.	0.	-2.000	0.	-1.000				
1.000	1.000			0.	1.414				

Q=(0,1,0) FOR BODY NO. 2 OF SURFACE NO. 10

0.

COEFFICIENTS FOR BODY NO. 2 OF SURFACE NO. 10

C=	0.	D=	0.	E=	0.	A=	0.	B=	0.
G=	0.	H=	0.099999999E 01	I=	-0.099999999E 01	F=	0.	J=	-0.
1	10	3	0.	0.	1.000				
2.000	0.	0.	-2.000	0.	0.				
1.000	-1.000				1.414				

Q=(0,1,2) FOR BODY NO. 3 OF SURFACE NO. 10

0.

COEFFICIENTS FOR BODY NO. 3 OF SURFACE NO. 10

C=	0.	D=	0.	E=	0.	A=	-0.	B=	-0.
G=	0.	H=	0.099999999E 01	I=	-0.099999999E 01	F=	-0.	J=	-0.
7	11	5	-0.	-0.	-0.				



TABLE 6-7 (Continued)

SURFACE NUMBER	1	1	2	-0	-0
SURFACE NUMBER	2	2	WAS PUT ON TAPE		
SURFACE NUMBER	3	3	WAS PUT ON TAPE	-0	-0
SURFACE NUMBER	4	4	WAS PUT ON TAPE	-0	-0
SURFACE NUMBER	5	5	WAS PUT ON TAPE		
SURFACE NUMBER	6	6	WAS PUT ON TAPE	-0	-0
SURFACE NUMBER	7	7	WAS PUT ON TAPE		
SURFACE NUMBER	8	8	WAS PUT ON TAPE		
SURFACE NUMBER	9	9	WAS PUT ON TAPE	-0	-0
SURFACE NUMBER	10	10	WAS PUT ON TAPE	-0	-0
 DELS	 S= 1	 T= 2	 A(S)F(S,T)=	 0.31999999E-00	 0.44799999E-00
				0.46705993E 01	0.44799999E-00
	S= 1	AREA (S)=	0.23039999E 02		
	S= 1	T= 2	F(S,T)=	0.20310761E-00	
				-0	-0
 DELS	 S= 1	 T= 3	 A(S)F(S,T)=	 0.31999999E-00	 0.25600000E-00
				0.48154933E 01	0.31999999E-00
	S= 1	AREA (S)=	0.23039999E 02		
	S= 1	T= 3	F(S,T)=	0.20905589E-00	
				-0	-0
 DELS	 S= 1	 T= 4	 A(S)F(S,T)=	 0.31999999E-00	 0.19199999E-00
				0.46101402E 01	0.31999999E-00
	S= 1	AREA (S)=	0.23039999E 02		
	S= 1	T= 4	F(S,T)=	0.20009290E-00	
				-0	-0



TABLE 6-7 (Continued)

$S= 1$	$T= 1$	$F(S,T) = 0.44799999E-00$	$A(S,F(S,T)) = 0.44799999E-00$	$0.25600000E-00$	$0.31999999E-00$
DELS					
$S= 2$	$T= 1$	$A(S,F(S,T)) = 0.46705992E-01$			
DELS					
$S= 2$	$AREA (S) =$	$0.22880255E-02$			
DELS					
$S= 2$	$T= 1$	$F(S,T) = 0.2052565E-00$			
DELS					
i_1	i_1	i_2	i_1	i_2	-0
$S= 1$	$T= 2$	$F(S,T) = 0.25600000E-00$	$A(S,F(S,T)) = 0.31999999E-00$	$0.44799999E-00$	$0.31999999E-00$
DELS					
$S= 1$	$T= 2$	$A(S,F(S,T)) = 0.35802703E-01$			
DELS					
$S= 1$	$AREA (S) =$	$0.23039999E-02$			
DELS					
$S= 1$	$T= 2$	$F(S,T) = 0.1559367E-00$			
DELS					
i_1	i_1	i_3	i_1	i_3	-0
$S= 1$	$T= 3$	$F(S,T) = 0.25600000E-00$	$A(S,F(S,T)) = 0.31999999E-00$	$0.31999999E-00$	$0.25600000E-00$
DELS					
$S= 1$	$T= 3$	$A(S,F(S,T)) = 0.37014874E-01$			
DELS					
$S= 1$	$AREA (S) =$	$0.23039999E-02$			
DELS					
$S= 1$	$T= 3$	$F(S,T) = 0.16065484E-00$			
DELS					
i_1	i_1	i_4	i_1	i_4	-0
$S= 1$	$T= 4$	$F(S,T) = 0.25600000E-00$	$A(S,F(S,T)) = 0.31999999E-00$	$0.15199999E-00$	$0.31999999E-00$
DELS					
$S= 1$	$T= 4$	$A(S,F(S,T)) = 0.66031986E-00$			
DELS					
$S= 1$	$AREA (S) =$	$0.23039999E-02$			
DELS					
$S= 1$	$T= 4$	$F(S,T) = 0.28659717E-01$			
DELS					
i_1	i_1	i_5	i_1	i_5	-0
DELS					
$S= 1$	$T= 5$	$A(S,F(S,T)) = 0.79162097E-01$			
DELS					
$S= 1$	$AREA (S) =$	$0.23039999E-02$			
DELS					
$S= 1$	$T= 5$	$F(S,T) = 0.3436358E-00$			
DELS					
i_1	i_5	i_1	i_5	i_1	-0



TABLE 6-7 (Continued)

DELS	0.26666667E-00	0.26666667E-00	0.25600000E-00	0.31999999E-00
S= 5	T= 1	A(S)F(S,T)=	0.79180190E 01	
		AREA (S)=	0.44474768E 02	
S= 5	T= 1	F(S,T)=	0.17803373E-00	
1	6		-0	-0
DELS	0.26666667E-00	0.26666667E-00	0.25600000E-00	0.31999999E-00
S= 5	T= 1	A(S)F(S,T)=	0.69512165E 01	
		AREA (S)=	0.44474768E 02	
S= 6	T= 1	F(S,T)=	0.15629573E-00	
1	7		-0	-0
DELS	0.18856186E-00	0.18856186E-00	0.25600000E-00	0.31999999E-00
S= 7	T= 1	A(S)F(S,T)=	0.38166459E 01	
		AREA (S)=	0.14741064E 02	
S= 7	T= 1	F(S,T)=	0.25889846E-00	
1	8		-0	-0
DELS	0.18856186E-00	0.18856186E-00	0.25600000E-00	0.31999999E-00
S= 8	T= 1	A(S)F(S,T)=	0.31685018E 01	
		AREA (S)=	0.17798078E 02	
S= 8	T= 1	F(S,T)=	0.17802998E-00	
1	9		-0	-0
DELS	0.18856186E-00	0.18856186E-00	0.25600000E-00	0.31999999E-00
S= 9	T= 1	A(S)F(S,T)=	0.14001546E 01	
		AREA (S)=	0.17798078E 02	
S= 9	T= 1	F(S,T)=	0.78668863E-01	
1	5		-0	-0
DELS	0.26666667E-00	0.26666667E-00	0.44799999E-00	0.44799999E-00



TABLE 6-7 (Continued)

S= 5	S= 5	T= 2	A(S,F(S,T))=	0.68297503E 01
S= 5	AREA (S)=		0.44474768E 02	
S= 1	T= 2	6	F(S,T)=	0.15356460E-00
				-0
				-0
DElS		0.26666557E-00	0.26666667E-00	0.44799999E-00
S= 5	T= 2	A(S,F(S,T))=	0.82897630E 01	0.44799999E-00
S= 5	AREA (S)=		0.44474768E 02	
S= 6	T= 2	5	F(S,T)=	0.18639249E-00
				-0
				-0
n.	3	1	-1.000	
0.	0.			
I= 1	S= 5	T= 1	P(I)F(S,T)=	0.53672434E-02
POINT ON S IS X(T))=			Y(T))=	0.
I= 2	S= 5	T= 1	P(I)F(S,T)=	0.53672446E-02
POINT ON S IS X(T))=			Y(T))=	0.
				-0
n.	2	1	-1.000	
0.	0.			
I= 1	S= 5	T= 1	P(I)F(S,T)=	0. Y(T))=
POINT ON S IS X(T))=			Y(T))=	-0.20000000E 01
I= 2	S= 5	T= 1	P(I)F(S,T)=	0.106954E-00
POINT ON S IS X(T))=			Y(T))=	0.20000000E 01
				-0
n.	1	0.	-1.000	
0.	0.			
I= 1	S= 5	T= 1	P(I)F(S,T)=	0. Y(T))=
POINT ON S IS X(T))=			Y(T))=	-0.20000000E 01
I= 2	S= 5	T= 1	P(I)F(S,T)=	0. Y(T))=
POINT ON S IS X(T))=			Y(T))=	-0.16000000E 01
I= 3	S= 5	T= 1	P(I)F(S,T)=	0.65256266E 00
POINT ON S IS X(T))=			Y(T))=	0.
I= 4	S= 5	T= 1	P(I)F(S,T)=	0.983641E 00
POINT ON S IS X(T))=			Y(T))=	0.12000000E 01
				-0
				-0



TABLE 6-7 (Continued)

S = 10 ⁻⁴	AREA (S) = -0	0.88990434E 01 -0 -0
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TABLE 6-8
SUMMARY OF EXAMPLE PROBLEM #3

From Surface s	To Surface t	AF Product Computed/True	Area of Surface s Computed/True	Shape Factor Computed/True
Top S(2)	Front S(1)	4.680	22.88	.2045
		4.608	23.04	.2000
Front S(1)	Top S(2)	4.680	23.04	.2031
		4.608	23.04	.2000
Front S(1)	Side S(3)	4.815	23.04	.2090
		4.608	23.04	.2000
Front S(1)	Back S(4)	4.610	23.04	.2001
		4.608	23.04	.2000
Front S(1)	Sphere-X	7.918	23.04	.3437
		8.377	23.04	.3636
Front S(1)	Sphere-Z	6.951	23.04	.3017
		8.377	23.04	.3636
Front S(1)	Sphere-P	8.385	23.04	.3639
		8.377	23.04	.3636
Sphere - P	Front S(1)	8.385	50.34	.1666
		8.377	50.26	.1667
Sphere - X	Front S(1)	7.918	44.47	.1780
		8.377	50.26	.1667
Sphere - Z	Front S(1)	6.951	44.47	.1563
		8.377	50.26	.1667
Sphere - X	Top S(2)	6.830	44.47	.1536
		8.377	50.26	.1667
Sphere - Z	Top S(2)	8.290	44.47	.1864
		8.377	50.26	.1667
Front S(1)	Top S(2)	3.58	23.04	.155
With Interference by S(5)		3.50	23.04	.152 *
Front S(1)	Side S(3)	3.70	23.04	.161
With Interference by S(5)		3.50	23.04	.152 *
Front S(1)	Back S(4)	.660	23.04	.029
With Interference by S(5)		.660	23.04	.029 *

*These "true" values are computed on the assumption that since F(1,4) is correct, F(1,4) with interference by S(5) is correct.

$S(5)$, the sphere with x -dependent, $S(6)$, the sphere with z -dependent, and the values computed for the entire sphere from the AF products and areas computed for the subsurfaces $S(7)$, $S(8)$ and $S(9)$ of the partitioned sphere.

The true values used for the three shape factors with interference are based on the assumption that, since the AF product and shape factor from the front surface to the back without interference, $F(1,4)$, is correct to three significant figures, the value of the AF product and $F(1,4)$ with interference will be correct to three significant figures. Therefore, since $F(1,4) = .029$ with interference and the sum of the five shape factors from $S(1)$ to the sides must equal .636, the shape factors $F(1,2)$ and $F(1,3)$ should be $(.636 - .029)/4 = .152$ and the AF products $.152 \times 23.04 = 3.50$.

Comparing the areas, the computed area of $S(2)$ is .7% low, while the area of $S(1)$ is exact. This is due to the fact that $S(1)$ covers the rectangle of the limits on the dependent coordinates exactly, while $S(2)$ does not. Any plane area such as $S(1)$, $S(3)$, and $S(4)$ which covers the rectangle of the dependent coordinates is computed exactly regardless of the number of grid points used. The error on $S(2)$ is caused by points whose differential areas overlap the edge of the surface. Note that if the majority of these points happened to be inside the square $S(2)$, their areas would be included and the area would have been too large rather than too small.

This difficulty can never be overcome completely as long as the surface does not cover the rectangle of the dependent coordinates. However, if the surface is partitioned into subareas with the appropriate rectangle of independent coordinates, the number of grid points can be increased for a resultant increase in accuracy. On the sphere the areas computed on $S(5)$ and $S(6)$ are affected by the same condition as the cone $S(1)$, the sphere $S(5)$, and the cylinder $S(9)$ in example problem #2. By partitioning the sphere into three subareas, the error is decreased from 11.4% to .16%.

In comparing the AF products and shape factors for the cube, it is noted that the factor $F(1,4)$ to the opposite side is exact while the factor to the top and adjacent sides are too high. This is entirely due to the areas very close to the common edge. This inaccuracy can be reduced, but never entirely eliminated, by partitioning.



If the subareas are concentrated along the common boundary, the partitioning gives the most improvement for the least cost. Note on the AF products the differences between computed and actual for both $F(1,3)$ with and $F(1,3)$ without interference by the sphere are almost identical. While these differences for $F(1,2)$ with and without interference are very close to the same value. This indicates that the error is principally confined to the area near the common boundary.

The AF products and shape factors from sphere to plane show the effect of the choice of dependent variable as well as the advantage of partitioning. The effect of the area on the plane is shown by the comparison of the values computed for the front and the top. With the accurately computed front and x -dependent (the best choice) the AF product shows an error of -5.5% and the shape factor an error of $+6.8\%$. If the z is dependent (the worst choice), these errors become -17% and -6.3% , respectively, while with the partitioned sphere these errors are reduced to $+.05\%$ and $-.06\%$.

Comparing the AF product from sphere $-X$ to the top and from sphere $-Z$ to the front, the difference simply mirrors the low area computed for the top. The size 8.29 of the AF product from sphere $-Z$ to the top is explained by the fact that the top $(0,0,2)$ of the sphere where the grid points are most densely packed is also the point closest to the top of the cube, while on sphere $-X$ the densest collection of grid points is around $(2,0,0)$ while the point closest to the front of the cube is $(1.6,1.2,0)$. Thus the effect of the reduced area for $S(2)$ (the top) on the AF product is relatively small. The point-to-surface shape factors show the variation in importance of the differential areas as the point moves away from the point $(1.6,1.2,0)$ where the sphere and the front are the closest. At $(1.6, 1.2, 0)$ the factor is .984 at $(2,0,0)$ the value drops to .652, and at $(0,2,0)$ the factor is only .319. At the top of the sphere at $(0,0,2)$ the value has dropped to only .005. These values show the importance of concentrating the grid points near the point where the distance between the surfaces is a minimum.

Taken as a whole, this problem illustrates most of the factors which affect accuracy. Additional cases such as from the bottom to the top or from the partitioned sphere to the top could have been computed, however these



results are easily predicted from the work already done. For the shape factor from bottom to top the excluded areas would decrease the AF product, but the shape factor would be nearly correct. In the case of the AF product and shape factor from the partitioned sphere to the top, the AF product would be slightly smaller which would decrease the shape factor and result in a slightly more accurate AF product but less accurate shape factor.

As noted in the boundary input section, page 6-62, the boundary B(2,1) composed of intersecting planes is an improper boundary for this surface since there is an unbounded area in the opposite angle of the double plane cylinder. Under the conditions imposed by the geometry of S(2) one extraneous grid point is produced. As a result the area of S(2) includes an area which is not a part of the cube. The size of this area A' can be computed from the limits on the independent variables, GS and the angle α between the z axis and the normal to S(2).

$$\begin{aligned} A' &= (x_{\max} - x_{\min})(y_{\max} - y_{\min}) \cos(\alpha) / (GS)^2 \\ &= 6.72 \times 6.72 \times \cos(0) / 15^2 \\ &= .200704 \end{aligned}$$

Thus the area of S(2) should be 22.68 instead of 22.83, the corresponding error 1.6% rather than .7%. The distinction between the use of two intersecting planes as a double plane cylinder vs the use of the same planes as two distinct boundaries is also illustrated by the areas 17.8 and 3.9 computed for S(9) and S(10) respectively. In partitioning the sphere, S(9) is defined by using a double plane cylinder of intersecting planes as a boundary. Thus, S(9) includes two opposed sections of the sphere. For S(10), the two planes are used as separate boundaries. As shown in Figure 6-16, S(10) consists of a single section or one-half of S(9). These examples were used to emphasise the fact that a boundary cylinder formed by two intersecting planes splits the surface into four sections which are grouped in two sets of opposed sections. This is very useful for the positioning of cones, cylinders, spheres etc. as shown in Example problems 2 and 3. Two additional surfaces would be required for each of these if single plane boundaries were used in place of the double plane cylinders. Note that for the spheres, a boundary cone has a similar property. However it has also been shown that uncritical

use of this type of boundary can lead to error. In general the cylinder of intersecting planes should be used only when areas in opposed sections are required.

In this problem relatively small values of GS were used to save computer time without a serious loss of accuracy. Since computer time varies directly as the product of $[GS(1) \times GS(2)]^2$, doubling the value of GS on one surface increases the computing time by a factor of 4, while doubling GS on both surfaces increases the computing time by a factor of 16. Increasing GS to 20 for the sides of the cube would increase the computer time by a factor of $(20/15)^4 = (4/3)^4 = 3.16$. A partitioning scheme which concentrates the grid points along the common boundary would be more economical than an increase of GS on the unpartitioned surface. For example each plane could be split into a half and two quarters with the quarters along the common boundary. With GS = 10 on each of the subsurfaces, the accuracy will equal that given with GS = 20 on the unpartitioned surfaces. Without utilizing the symmetry of the problem, this will reduce the computer time factor from 3.16 to $9(10/15)^4 = 16/9 = 1.78$. Utilizing symmetry gives $4 \times (10/15)^4 = .79$ or a 21% saving in computer time. Alternately the GS on these surfaces could be increased for further accuracy for the same computer time as the single surfaces with GS = 15. The shape factor from front to side would be reduced from .2090 to .2068, or a reduction from 4.5% error to 3.4% error. The improvement in the factors to the top and back will be less since the error for GS = 15 is lower. In computations involving the partitioned sphere the change would be less than .1% at a cost of a 78% increase in computer time, assuming GS = 15 for each section of the sphere.

APPENDIX A

DERIVATION OF SURFACE INPUT EQUATIONS

This appendix describes the translation of the program input data into coefficients of the equations for the given surfaces. Equations are derived for each surface type in the order listed in Section III. As a preliminary, two subroutines are considered. The first is concerned with the solution of simultaneous homogeneous linear equations, while the other provides equations for the transformation of coordinate systems.

In the routines for computing surface equations, the coefficients frequently occur as the solution of simultaneous homogeneous linear equations. Assume a system of n equations in the variables V_j , $j = 1, \dots, (n + 1)$ represented by:

$$a_{11} V_1 + a_{12} V_2 + \dots + a_{1(n+1)} V_{(n+1)} = 0$$

$$a_{21} V_1 + a_{22} V_2 + \dots + a_{2(n+1)} V_{(n+1)} = 0$$

⋮

⋮

$$a_{n1} V_1 + a_{n2} V_2 + \dots + a_{n(n+1)} V_{(n+1)} = 0$$

Let $[a_{ij}]$ represent the matrix of coefficients of this system. Since there are n equations in $n + 1$ variables, the matrix is rectangular of size n by $(n + 1)$. Thus, the removal of one column reduces this matrix to a square matrix. Let $[a_{ij}]_k$ represent the square matrix obtained by deleting the k -th column of the matrix $[a_{ij}]$, and let $\det [a_{ij}]_k$ represent the determinant of



$[a_{ij}]_k$. In this notation, the solution of the system of equations used by this program is given by the equation:

$$v_k = (-1)^{(k+1)} \left(\det [a_{ij}]_k \right) \quad k = 1, \dots, (n+1).$$

The principal value of this solution is that it reduces to the trivial solution $v_k = 0$ for all $k = 1, \dots, (n+1)$ whenever the input data are incapable of defining a single surface. For example, if four coplanar points are chosen to define a sphere and the four points are on the circumference of a circle, there are an infinite number of spheres through the points. If the points are not on a circle, there is no sphere containing the four points. Or consider three colinear points. The general solution to the equations related to this problem has to include the equations of all the planes passing through these points. In either of these cases, the program gives the trivial solution $v_k = 0$ for all $k = 1, \dots, (n+1)$. This has a double effect on the program operation. First, it cancels all computations requiring this surface, thereby saving computing time. And secondly, it provides a flag to show that the surface is inadequately defined by the input.

As an example of the difference in the equation of a surface vs. the equation of a family of surfaces, consider the plane through the points $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 4)$, as well as the family of planes through $(2, 1, 0)$, $(2, 1, 3)$, and $(2, 1, 17)$. The equation of the plane in intercept form is:

$$x + y/2 + z/4 = 1$$

However, from this it can be seen that for any non-zero value C of the coefficient of z , the plane will also be defined by the equation $4Cx + 2Cy + Cz - 4C = 0$.

Note, that in effect, the other three coefficients are written as linear functions of the single coefficient C . This is characteristic of the equation of a surface, that the coefficients may be expressed as linear functions of any of the non-zero coefficients with any non-zero value of this



independent coefficient giving a set of coefficients for the same surface.
On the other hand, the equation of the family of surfaces is:

$$Ax + By - (2A + B) = 0$$

In this equation there are two independent coefficients, hence no two can be written in terms of the third. This is also characteristic of families of surfaces. Thus, it can be seen that if the general solution V_1, \dots, V_{n+1} of the n simultaneous homogeneous equations is to define a surface rather than a family of surfaces, it must be possible to write each of the variables V_k as a linear function of any non-zero variable V_h .

A variable is called a non-zero variable if there exists a solution in which it is non-zero. Assume that V_h is a non-zero variable and write the system of equations in the form:

$$\begin{aligned} a_{11} V_1 + \dots + a_{1(h-1)} V_{h-1} + a_{1(h+1)} V_{h+1} + \dots + a_{1(n+1)} V_{n+1} &= -a_{1h} V_h \\ \vdots &\vdots \\ a_{nl} V_1 + \dots + a_{n(h-1)} V_{h-1} + a_{n(h+1)} V_{h+1} + \dots + a_{n(n+1)} V_{n+1} &= -a_{nh} V_h \end{aligned}$$

From the preceding discussion, it can be seen that if the variables V_1, \dots, V_{n+1} are to be the coefficients of the equation of a surface, these equations considered as n non-homogeneous equations in the n variables $V_1, \dots, V_{h-1}, V_{h+1}, \dots, V_{n+1}$ must have a unique solution in terms of V_h . But this means that $\det [a_{ij}]_h$ is not zero and Cramer's rule can be applied. However, this rule is modified here so that the determinant in the numerator for the unknown V_k is $\det [a_{ij}]_k$. This is accomplished by factoring $-V_h$ from, and rearranging the columns of, the standard numerator determinant. As a result, the variable V_k if $k \neq h$ is given by:

$$V_k = (-1)^{(k-h)} V_h \left(\det [a_{ij}]_k \right) / \left(\det [a_{ij}]_h \right)$$



If V_h is now given the value $(-1)^{(h+1)} \left(\det [a_{ij}]_h \right)$, the equation for V_k reduces to:

$$V_k = (-1)^{k+1} \left(\det [a_{ij}]_k \right)$$

Note that the expression for V_k is independent of h and that if $k = h$ the equation gives V_h its own assigned value. Hence, on the assumption that there exists a $k = h$ so that $\det [a_{ij}]_h \neq 0$, the equation for V_k holds for all $k = 1, \dots, n+1$. Tracing this assumption back through the previous reasoning shows that this assumption implies that the non-homogeneous equations have a unique solution. Hence, the variables can be written as unique linear functions of a non-zero variable V_h , which implies that the variables V_1, \dots, V_{n+1} are the coefficients of an equation of a surface. On the other hand, if $\det [a_{ij}]_k = 0$ for all $k = 1, \dots, n+1$, the non-homogeneous equations have no solution. Hence, the variables cannot be written as unique linear functions of a non-zero variable and V_1, \dots, V_{n+1} are the coefficients of a family of surfaces. This demonstrates that the expression for V_k gives the desired solution.

In several of the formats, the equations are first computed in terms of a translated and rotated coordinate system x'', y'', z'' . In each such input format, the point which is to be the origin in this transformed system is called P_0 with coordinates (x_0, y_0, z_0) . However, since the cases differ, this routine is entered at different steps in the various cases. To facilitate the indexing throughout, the direction cosines of the x'' axis are called λ, μ, ν , of the y'' axis λ_1, μ_1, ν_1 , and of the z'' axis λ_2, μ_2, ν_2 . Using this notation, a transformation routine is developed to reduce the equation $A''x''^2 + B''y''^2 + C''z''^2 + 2G''x'' + J'' = 0$ to an equation in x, y and z . The equations of rotation relating x'', y'', z'' to axes x', y', z' through P_0 but parallel to x, y, z are:

$$x'' = \lambda x' + \mu y' + \nu z'$$

$$y'' = \lambda_1 x' + \mu_1 y' + \nu_1 z'$$

$$z'' = \lambda_2 x' + \mu_2 y' + \nu_2 z'$$



Substitution gives:

$$A'' (\lambda x' + \mu y' + \nu z')^2 + B'' [(\lambda_1 x' + \mu_1 y' + \nu_1 z')^2 + (\lambda_2 x' + \mu_2 y' + \nu_2 z')^2] \\ + 2G'' (\lambda x' + \mu y' + \nu z') + J'' = 0$$

Squaring and grouping in powers of the variables gives:

$$[A''\lambda^2 + B''(\lambda_1^2 + \lambda_2^2)]x'^2 + [A''\mu^2 + B''(\mu_1^2 + \mu_2^2)]y'^2 \\ + [A''\nu^2 + B''(\nu_1^2 + \nu_2^2)]z'^2 + 2[A''\lambda\mu + B''(\lambda_1\mu_1 + \lambda_2\mu_2)]x'y' \\ + 2[A''\lambda\nu + B''(\lambda_1\nu_1 + \lambda_2\nu_2)]x'z' + \\ + 2[A''\mu\nu + B''(\mu_1\nu_1 + \mu_2\nu_2)]y'z' + 2G''\lambda x' + 2G''\mu y' \\ + 2G''\nu z' + J'' = 0$$

Substituting for λ_1, μ_1, ν_1 and λ_2, μ_2, ν_2 gives:

$$[B'' + (A'' - B'')\lambda^2]x'^2 + [B'' + (A'' - B'')\mu^2]y'^2 + [B'' + (A'' - B'')\nu^2]z'^2 \\ + 2(A'' - B'')\lambda\mu x'y' + 2(A'' - B'')\lambda\nu x'z' + 2(A'' - B'')\mu\nu y'z' \\ + 2G''\lambda x' + 2G''\mu y' + 2G''\nu z' + J'' = 0$$

Which is an equation of the form

$$A'x'^2 + B'y'^2 + C'z'^2 + 2D'x'y' + 2E'x'z' + 2F'y'z' + 2G'x' + 2H'y' \\ + 2I'z' + J' = 0$$

Translation to the x, y, z axes is accomplished by means of the equations:

$$x' = x - x_o$$

$$y' = y - y_o$$

$$z' = z - z_o$$



Substituting for x' , y' , and z' gives:

$$\begin{aligned}
 A' (x - x_0)^2 + B' (y - y_0)^2 + C' (z - z_0)^2 + 2D' (x - x_0) (y - y_0) \\
 + 2E' (x - x_0) (z - z_0) + 2F' (y - y_0) (z - z_0) \\
 + 2G' (x - x_0) + 2H' (y - y_0) + 2I' (z - z_0) + J' = 0
 \end{aligned}$$

Clearing parentheses and collecting in powers of the variables gives:

$$\begin{aligned}
 A'x^2 + B'y^2 + C'z^2 + 2D'xy + 2E'xz + 2F'yz + 2F'yz \\
 + 2 \left[G' - (A'x_0 + D'y_0 + E'z_0) \right] x + 2 \left[H' - (D'x_0 + B'y_0 + F'z_0) \right] y \\
 + 2 \left[I' - (E'x_0 + F'y_0 + C'z_0) \right] z + \left\{ J' + A'x_0^2 + B'y_0^2 + C'z_0^2 \right. \\
 \left. + 2 \left[D'x_0y_0 + E'x_0z_0 + F'y_0z_0 - (G'x_0 + H'y_0 + I'z_0) \right] \right\} = 0
 \end{aligned}$$

Assembling this information gives the following equations for the general quadric in x , y and z . These equations comprise the transformation subroutine which is used entirely or in part to transform the equations into the x , y , z coordinate system. This routine transforms only equations of the form $A''x''^2 + B''y''^2 + B''z''^2 + 2G''x'' + J'' = 0$ into the equivalent equation in the x , y , z system. However, parts of this routine can be used for the transformation of equations of slightly different form. Therefore, three entry points for this routine are identified. No alternate exits have been provided as there is no use for partially transformed equations since the program requires that every equation be in the same x , y , z coordinate system.



Entry #1 for the complete transformation routine:

$$G' = G' \lambda$$

$$H' = G'' \mu$$

$$I' = G'' \nu$$

$$K' = A'' - B''$$

Entry #2 if G' , H' , I' and K' are known or must be computed independently:

$$A = B'' + K' \lambda^2 \quad D = K' \lambda \mu$$

$$B = B'' + K' \mu^2 \quad E = K' \lambda \nu$$

$$C = B'' + K' \nu^2 \quad F = K' \mu \nu$$

Entry #3 if translation only is required:

$$G = G' - (Ax_o + Dy_o + Ez_o)$$

$$H = H' - (Dx_o + By_o + Fz_o)$$

$$I = I' - (Ex_o + Fy_o + Cz_o)$$

$$J = J'' + Ax_o^2 + By_o^2 + Cz_o^2 + 2 \left[Dx_o y_o + Ex_o z_o + Fy_o z_o \right]$$

$$- \left(G' x_o + H' y_o + I' z_o \right)$$

Considering the surface equation computations in the order given in the format table, the first input format requiring computation is the plane given three points. For a plane, the quadric equation $\theta(x, y, z) = 0$ reduces to the linear equation

$$2Gx + 2Hy + 2Iz + J = 0$$

Substituting the coordinates of each of the three input surface points gives the simultaneous homogeneous linear equations:

$$2x_1G + 2y_1H + 2z_1I + J = 0$$

$$2x_2G + 2y_2H + 2z_2I + J = 0$$

$$2x_3G + 2y_3H + 2z_3I + J = 0$$

These equations can be solved by the general solution already developed. However, since the coefficient of J in the first equation is +1, hence never zero, this solution can be shortened by solving for G , H and I by the general solution and then obtaining J from the equation:

$$J = -2(Gx_1 + Hy_1 + Iz_1).$$

If the center $P_o = (x_o, y_o, z_o)$ and the radius (R) are known, the equation of the sphere is

$$x^2 + y^2 + z^2 - 2x_o x - 2y_o y - 2z_o z + (x_o^2 + y_o^2 + z_o^2 - R^2) = 0$$

Hence

$$A = B = C = 1, D = E = F = 0, G = -x_o, H = -y_o, I = -z_o$$

and $J = x_o^2 + y_o^2 + z_o^2 - R^2$ are the coefficients.

If four points $P_i = (x_i, y_i, z_i)$ $i = 1, \dots, 4$ on a sphere are given, substitution into the standard equation of the sphere gives the four linear simultaneous equations:

$$2x_iG + 2y_iH + 2z_iI + J = -(x_i^2 + y_i^2 + z_i^2) \quad i = 1, \dots, 4$$

If $u_i = \sqrt{x_i^2 + y_i^2 + z_i^2}$ then Cramer's rule gives the solution for G, H and I:

$$u = 2 \det [x_i, y_i, z_i, 1] \quad i = 1, \dots, 4$$

$$G = (\det [u_i, y_i, z_i, 1]) / u \quad i = 1, \dots, 4$$

$$H = (\det [x_i, u_i, z_i, 1]) / u \quad i = 1, \dots, 4$$

$$I = (\det [x_i, y_i, u_i, 1]) / u \quad i = 1, \dots, 4$$

While J is given by:

$$J = u_1 - 2(Gx_1 + Hy_1 + Iz_1).$$

A right circular cylinder with axis along the x'' axis has an equation of the form $y''^2 + z''^2 - R^2 = 0$. Therefore, given two axis points (P_o, P_1) and the radius (R) of the cylinder, the direction numbers of the axis will be:

$$l = x_1 - x_o$$

$$m = y_1 - y_o$$

$$n = z_1 - z_o$$

Then the direction cosines of the axis may be obtained from the equations:

$$d = (l^2 + m^2 + n^2)^{1/2}$$

$$\lambda = l/d$$

$$\mu = m/d$$

$$\nu = n/d$$



Since $G'' = 0$, the coefficients G' , H' , and I' will also be zero. Thus, computing time can be saved by settling $G' = H' = I' = 0$, $B'' = 1$, $K' = -1$, $J'' = -R^2$ and using the second entry to the transformation routine.

In the case of the right circular cylinder given the direction numbers $(l: m: n)$ of an element plus three points (P_0, P_1, P_2) the first step is to establish a coordinate system.

The point P_0 is taken as the origin with $(l: m: n)$ as direction numbers of the positive x'' axis. Now at least one of the numbers l , m , and n must be non-zero. Therefore, starting with n each will be tested. If n is not zero, the direction numbers of the y'' axis are computed by:

$$l_1 = (x_1 - x_0)$$

$$m_1 = (y_1 - y_0)$$

$$n_1 = -(l_{11} + mn_1)/n$$

If $n = 0$ but $m \neq 0$ then the direction numbers are given by:

$$l_1 = (x_1 - x_0)$$

$$m_1 = -ll_1/m$$

$$n_1 = (z_1 - z_0)$$

Finally, if $n = m = 0$, set $l_1 = 0$ and $m_1 = (y_1 - y_0)$

$$n_1 = (z_1 - z_0)$$

In any case, the direction numbers of the x'' and y'' axes can be turned into their respective direction cosines as in the preceding case. The direction cosines of the z'' axis are computed by the equations:

$$\lambda_2 = \mu\nu_1 - \nu\mu_1$$

$$\mu_2 = \nu\lambda_1 - \lambda\nu_1$$

$$\nu_2 = \lambda\mu_1 - \mu\lambda_1$$

Now the y'' and z'' coordinates of the points P_1 and P_2 can be computed by the equations:

$$y''_i = \lambda_1 (x_i - x_o) + \mu_1 (y_i - y_o) + \nu_1 (z_i - z_o) \quad i = 1, 2$$

$$z''_i = \lambda_2 (x_i - x_o) + \mu_2 (y_i - y_o) + \nu_2 (z_i - z_o) \quad i = 1, 2$$

Note that the transformed coordinates of P_o are $(0, 0, 0)$ and that for a cylinder with elements parallel to the x'' axis the x'' coordinates are unnecessary. Since the cylinder passes through P_o , the term J'' is zero and the equation is:

$$y''^2 + z''^2 + 2H''y + 2I''z = 0$$

Substituting the coordinates of P_1 and P_2 gives:

$$2y''_i H'' + 2z''_i I'' = - (y''_i^2 + z''_i^2) \quad i = 1, 2$$

The functions u , u_1 and u_2 will be defined by:

$$u = 2 (y''_1 z''_2 - z''_1 y''_2)$$

$$u_i = - (y''_i^2 + z''_i^2) \quad i = 1, 2$$



Then Cramer's rule gives H'' and I'' by the equations:

$$H'' = (u_1 z_2 - z_1 u_2)/u$$

$$I'' = (y_1 u_2 - u_1 y_2)/u$$

Since H'' and I'' are given rather than G'' , the intermediate rotated coefficients G' , H' and I' must be computed before entering the transformation routine. Since the rotation equations are homogeneous, the degree of a term is unchanged by rotation. Hence, only $H'' y'' + I'' z''$ must be considered. Substituting the rotated variables for y'' and z'' gives:

$$\begin{aligned} H''(\lambda_1 x' + \mu_1 y' + \nu_1 z') + I''(\lambda_2 x' + \mu_2 y' + \nu_2 z') &= (H''\lambda_1 + I''\lambda_2)x' \\ &+ (H''\mu_1 + I''\mu_2)y' + (H''\nu_1 + I''\nu_2)z' \end{aligned}$$

Thus the coefficients G' , H' and I' are given by:

$$G' = H''\lambda_1 + I''\lambda_2$$

$$H' = H''\mu_1 + I''\mu_2$$

$$I' = H''\nu_1 + I''\nu_2$$

The coefficients are then obtained by setting $B'' = 1$, $K' = -1$, $J'' = 0$ and using entry #2 of the transformation routine.

The computations for the arbitrary conic cylinder parallel those just given for the right circular cylinder using the direction numbers of an element and three surface points. The differences arise from the two extra surface points and the equation in the x'' , y'' , z'' system which is:

$$B'y''^2 + C'z''^2 + 2F'' y'' z'' + 2H''y'' + 2I''z'' = 0$$

The computation of the direction cosines of the axis system and the y''_i , z''_i coordinates of the surface points is accomplished using the equations developed for the right circular cylinder. However, for the surface points the index i will have a range from one to four rather than one and two. Substituting these points into the equation of the arbitrary conic cylinder gives four homogeneous simultaneous equations which can be solved by the general method:

$$y''_i^2 B'' + z''_i^2 C'' + 2y''_i z''_i F'' + 2 y''_i H'' + 2z''_i I'' = 0 \quad i = 1, \dots, 4$$



Since the equation of the arbitrary conic cylinder does not fit the format of the transformation routine, the rotation must be performed independently. Using the same equations of rotation, the equation is:

$$\begin{aligned} & B''(\lambda_1 x' + \mu_1 y' + \nu_1 z')^2 + C''(\lambda_2 x' + \mu_2 y' + \nu_2 z')^2 \\ & + 2F''(\lambda_1 x' + \mu_1 y' + \nu_1 z')(\lambda_2 x' + \mu_2 y' + \nu_2 z') \\ & + 2H''(\lambda_1 x' + \mu_1 y' + \nu_1 z') + 2I''(\lambda_2 x' + \mu_2 y' + \nu_2 z') = 0 \end{aligned}$$

Removing parentheses and collecting terms in powers of the variables gives the equation:

$$\begin{aligned} & (B''\lambda_1^2 + C''\lambda_2^2 + 2F''\lambda_1\lambda_2)x'^2 + (B''\mu_1^2 + C''\mu_2^2 + 2F''\mu_1\mu_2)y'^2 + \\ & (B''\nu_1^2 + C''\nu_2^2 + 2F''\nu_1\nu_2)z'^2 + 2[B''\lambda_1\mu_1 + C''\lambda_2\mu_2 + F''(\lambda_1\mu_2 + \mu_1\lambda_2)] \\ & x'y' + 2[B''\lambda_1\nu_1 + C''\lambda_2\nu_2 + F''(\lambda_1\nu_2 + \nu_1\lambda_2)]x'z' + 2[B''\mu_1\nu_1 + C''\mu_2\nu_2 \\ & F''(\mu_1\nu_2 + \nu_1\mu_2)]y'z' + 2(H''\lambda_1 + I''\lambda_2)x' + 2(H''\mu_1 + I''\mu_2)y' + \\ & 2(H''\nu_1 + I''\nu_2)z' = 0 \end{aligned}$$

Note that the coefficients G', H' and I' of x', y' and z' are those that were derived before. Thus the rotated equations are:

$$A = B''\lambda_1^2 + C''\lambda_2^2 + 2F''\lambda_1\lambda_2$$

$$B = B''\mu_1^2 + C''\mu_2^2 + 2F''\mu_1\mu_2$$

$$C = B''\nu_1^2 + C''\nu_2^2 + 2F''\nu_1\nu_2$$

$$D = B''\lambda_1\mu_1 + C''\lambda_2\mu_2 + F''(\lambda_1\mu_2 + \mu_1\lambda_2)$$

$$E = B''\lambda_1\nu_1 + C''\lambda_2\nu_2 + F''(\lambda_1\nu_2 + \nu_1\lambda_2)$$

$$F = B''\mu_1\nu_1 + C''\mu_2\nu_2 + F'' (\mu_1\nu_2 + \mu_2\nu_1)$$

$$G' = H''\lambda_1 + I''\lambda_2$$

$$H' = H''\mu_1 + I''\mu_2$$

$$I' = H''\nu_1 + I''\nu_2$$

The transformation routine from entry #3 gives G, H, I and J.

If the axis of a conic of revolution is taken as the x" axis, the equation of the conic of revolution is:

$$A''x''^2 + B''y''^2 + B''z''^2 + 2G''x'' + J'' = 0$$

Using the cylindrical coordinates (x'', ρ'', θ'') where ρ'' is distance from the x" axis and θ'' is the angle around the x" axis, this equation becomes:

$$A''x''^2 + B''\rho''^2 + 2G''x'' + J'' = 0$$

Note that since the surface is a body of revolution around the x" axis, this equation is independent of the angle θ . The direction numbers of the x" axis are:

$$l = x_4 - x_0$$

$$m = y_4 - y_0$$

$$n = z_4 - z_0$$

These are reduced to direction cosines by the method developed for the right circular cylinder. Hence the x" coordinates of the points

$P_i = (x_i, y_i, z_i) \quad i = 1, 2, 3$ are given by:

$$x''_i = \lambda(x_i - x_0) + \mu(y_i - y_0) + \nu(z_i - z_0)$$



If d_i is defined as the distance from P_o to P_i and α_i is defined as the angle $P_i P_o P_b$, the value of $\rho_i''^2$ $i = 1, 2, 3$ is given by:

$$\rho_i''^2 = d_i^2 \sin^2 \alpha_i = d_i^2 - d_i^2 \cos^2 \alpha_i \quad i = 1, 2, 3$$

The scalors l_i , m_i and n_i of the segment $P_o P_i$ $i = 1, 2, 3$ are given by:

$$l_i = x_i - x_o \quad i = 1, 2, 3$$

$$m_i = y_i - y_o \quad i = 1, 2, 3$$

$$n_i = z_i - z_o \quad i = 1, 2, 3$$

Hence d_i and $\cos \alpha_i$ are given by:

$$d_i^2 = l_i^2 + m_i^2 + n_i^2$$

$$\cos \alpha_i = (\lambda l_i + \mu m_i + \nu n_i) / d_i \quad i = 1, 2, 3$$

Hence $\rho_i''^2$ is given by:

$$\begin{aligned} \rho_i''^2 &= d_i^2 - (\lambda l_i + \mu m_i + \nu n_i)^2 = (1 - \lambda^2) l_i^2 + (1 - \mu^2) m_i^2 \\ &\quad + (1 - \nu^2) n_i^2 + 2[\lambda l_i (\mu m_i + \nu n_i) + \mu \nu m_i n_i] \end{aligned}$$

Substituting these coordinates into the equation of the conic in the cylindrical coordinates gives:

$$x_i''^2 A'' + \rho_i''^2 B'' + 2x_i'' G'' + J'' = 0 \quad i = 1, 2, 3$$



These simultaneous homogeneous equations can be solved for A'', B'' and G'' by the general method and for J'' by the shortcut:

$$J'' = - (A'' x_1''^2 + B'' \rho_1''^2 + 2G''x_1'')$$

The complete transformation routine will then be used to give the coefficients in the x, y, z system.

If nine surface points (P_1, \dots, P_9) are given, their coordinates may be substituted into the general quadratic equation giving nine linear homogeneous equations:

$$\begin{aligned} x_i^2 A + y_i^2 B + z_i^2 C + 2x_i y_i D + 2x_i z_i E + 2y_i z_i F + 2x_i G + 2y_i H \\ + 2z_i I + J = 0 \end{aligned}$$

These can be solved for A, ..., I by the general method; then -J is given by:

$$\begin{aligned} J = - \left\{ Ax_1^2 + By_1^2 + Cz_1^2 + 2 \left[(Dy_1 + Ez_1 + G)x_1 \right. \right. \\ \left. \left. + (Fz_1 + H)y_1 + Iz_1 \right] \right\} \end{aligned}$$

Since all boundaries are defined by the intersection of the working surface with quadric boundary surfaces, the surface equation computations are used wherever possible. In fact, all but one of the boundary surface types are handled in this manner. The one exception is the straight line boundary on a plane. In this case, the boundary surface is defined to be the plane through the given boundary line perpendicular to the given plane. Therefore, if $2G_1x + 2H_1y + 2I_1z + J_1 = 0$ is the equation of the working surface S_1 , and if P_0 and P_1 are points on the boundary, direction numbers of the normal to S_1 are G_1, H_1 and I_1 while direction numbers for the boundary line are:



$$l = x_1 - x_o$$

$$m = y_1 - y_o$$

$$n = z_1 - z_o$$

Hence, the coefficients G_b , H_b and I_b of the boundary surface must satisfy the equations:

$$lG_b + mH_b + nI_b = 0$$

$$G_l G_b + H_l H_b + I_l I_b = 0$$

These can be solved by the standard method and J_b can then be obtained from

$$J_b = -2(G_b x_o + H_b y_o + I_b z_o).$$



APPENDIX B

DERIVATION OF THE BASIC PROGRAM EQUATIONS

For quadric surfaces A_1 and A_2 , this program gives a solution for the shape factor equation:

$$F_{1,2} = \frac{1}{\pi A_1} \int_{A_1} \int_{A_2} \frac{Q_1 \cdot Q_2}{s^2} dA_2 \cdot dA_1$$

where Q_1 is the larger of the two quantities, $\cos \phi_1$ or zero and Q_2 is the larger of $\cos \phi_2$ or zero.

Let $\theta_1(x, y, z) = 0$ be the equation of A_1 and $\theta_2(x, y, z) = 0$ be the equation of A_2 where $\theta(x, y, z)$ represents the quadric form:

$$\theta(x, y, z) = Ax^2 + By^2 + Cz^2 + 2Dxy + 2Exz + 2Fyz + 2Gx + 2Hy + 2Iz + J$$

For convenience it is assumed that x is the dependent variable on A_1 , while z is dependent on A_2 . Any other choice of dependent variables can be handled by the program. However, since the derivations are exactly parallel, only the equations for the case of x dependent on A_1 and z dependent on A_2 are given. The machine sets up a square grid pattern in the plane of the independent coordinates; in the case of surface A_1 that would be the yz plane. This grid pattern gives the independent coordinates and the index of each grid point. Let $P_i(x_i, y_i, z_i)$ be a grid point on A_1 with independent coordinates, then $\theta_1(x_i, y_i, z_i) = 0$ can be solved for x .

$$\begin{aligned} A_1 x_i^2 + 2(D_1 y_i + E_1 z_i + G_1) x_i + [J_1 + B_1 y_i^2 \\ + 2(F_1 y_i z_i + H_1 y_i + I_1 z_i)] = 0 \end{aligned}$$

Each real solution of this equation gives the x coordinate of a grid point with independent coordinates y_i and z_i . Thus, the single pair (y_i, z_i) of

independent coordinates could give one, two, or no grid points. However, to avoid "vertical" grid points if $A \neq 0$, the machine rejects the solution when the quadratic has only one root.

Of course, if $A = 0$, the equation is linear and there is only one grid point for each pair of independent coordinates. Each grid point is then checked against the boundaries of the surface A_i with the computed grid points falling outside the specified boundaries being dropped and the remainder stored.

Let θ_{1x_i} , θ_{1y_i} and θ_{1z_i} represent the three partial derivations of $\theta_1(x, y, z)$ evaluated at the point P_i . These are given by:

$$\theta_{1x_i} = 2(A_{1x_i} + D_{1y_i} + E_{1z_i} + G_1)$$

$$\theta_{1y_i} = 2(D_{1x_i} + B_{1y_i} + F_{1z_i} + H_1)$$

$$\theta_{1z_i} = 2(E_{1x_i} + F_{1y_i} + C_{1z_i} + I_1)$$

Using this notation, the differential surface area dA_{li} of surface A_1 at point $P_i(x_i, y_i, z_i)$ is given in terms of its projection $dA_{li}^{(p)}$ on the plane of the independent variables (y and z) by the equation:

$$dA_{li} = \frac{\sqrt{\theta_{1x_i}^2 + \theta_{1y_i}^2 + \theta_{1z_i}^2}}{\theta_{1x_i}^2} dA_{li}^{(p)} = \frac{\sqrt{\theta_{1x_i}^2 + \theta_{1y_i}^2 + \theta_{1z_i}^2}}{|\theta_{1x_i}|} dA_{li}^{(p)}$$

Note that this equation is undefined if $\theta_{1x_i} = 0$; but this occurs only if the tangent plane at P_i is perpendicular to the yz-plane, hence only if P_i is a vertical point. Therefore, as was stated previously, if any vertical point is chosen as a grid point it must be replaced by four surrounding points. Note, that only the denominator of this expression is dependent on the identity of the dependent coordinate. Hence, the function T_{li} is defined by:

$$T_{li} = \sqrt{\theta_{1x_i}^2 + \theta_{1y_i}^2 + \theta_{1z_i}^2}$$

Thus, at any point, P_k , on any quadric surface, A_t , defined by $\theta_t(x, y, z) = 0$, if v represents the dependent coordinate on A_t , the relationship between dA_{tk} and the projected differential area $dA_{tk}^{(p)}$ is given by:

$$dA_{tk} = \frac{T_{tk}}{|\theta_t v|} dA_{tk}^{(p)}$$

Substituting these values into the shape factor equation gives:

$$F_{1,2} = \frac{1}{\pi A} \int_{A_1^{(p)}} \int_{A_2^{(p)}} \frac{Q_1 T_1 Q_2 T_2}{|\theta_{1x}| \cdot |\theta_{2z}| s^2} dA_2^{(p)} dA_1^{(p)}$$

Assuming that all the grid points of A_1 and A_2 have been computed and stored, the program computes an approximate value for this integral using point by point numerical integration. For each pair of grid points P_i on A_1 and P_j on A_2 , the program computes the integral V_{ij} of this equation:

$$V_{ij} = \frac{Q_{1ij} T_{1i} Q_{2ji} T_{2j}}{|\theta_{1x_i}| \cdot |\theta_{2z_j}| s_{ij}^2}$$

Now by definition s_{ij} is the distance from P_i to P_j . Therefore:

$$s_{ij}^2 = (x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2$$

The function Q_{1ij} was defined as the larger of the two values $\cos \phi_{1ij}$ or 0, where ϕ_{1ij} is defined as the angle between the normal to A_1 at point P_i and the line segment $P_i P_j$. A set of direction numbers for the surface A_1 at P_i will be $l_{1i} = \theta_{1x_i}$; $m_{1i} = \theta_{1y_i}$; $n_{1i} = \theta_{1z_i}$, while a set of direction numbers for $P_i P_j$ will be

$$l_{ij} = x_j - x_i; m_{ij} = y_j - y_i; n_{ij} = z_j - z_i$$

Hence the $\cos \phi_{lij}$ will be

$$\cos \phi_{lij} = \frac{\theta_{1x_i} (x_j - x_i) + \theta_{1y_i} (y_j - y_i) + \theta_{1z_i} (z_j - z_i)}{\sqrt{\theta_{1x_i}^2 + \theta_{1y_i}^2 + \theta_{1z_i}^2} \cdot \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}}$$

Similarly $\cos \phi_{2ji}$ is given by:

$$\cos \phi_{2ji} = \frac{\theta_{2x_j} (x_i - x_j) + \theta_{2y_j} (y_i - y_j) + \theta_{2z_j} (z_i - z_j)}{\sqrt{\theta_{2x_j}^2 + \theta_{2y_j}^2 + \theta_{2z_j}^2} \cdot \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}}$$

Since the denominator of the expression for $\cos \phi_{lij}$ is $T_{li} \cdot s_{ij}$, the value of $W_{lij} = s_{ij} T_{li}$, $\cos \phi_{lij}$ is:

$$W_{lij} = \theta_{1x_i} (x_j - x_i) + \theta_{1y_i} (y_j - y_i) + \theta_{1z_i} (z_j - z_i)$$

Now by definition, if P_i and P_j are distinct points, T_{li} and s_{ij} are positive, while if P_i and P_j are the same point, $(x_j - x_i) = (y_j - y_i) = z_j - z_i = 0$. Hence $\cos \phi_{lij}$ will be greater than zero if, and only if W_{lij} is greater than zero. Thus, ϕ_{lij} is greater than zero if and only if W_{lij} is positive. Since the numerator of V_{ij} is a product of factors, V_{ij} will be zero whenever any one of the factors is zero. Thus, if W_{lij} is less than or equal to zero, V_{ij} will be set equal to zero without further computation. If W_{lij} is positive, the corresponding factor $W_{2ji} = s_{ij} T_{2j} \cos \phi_{2ji}$ is computed by the formula:

$$W_{2ji} = \theta_{2x_j} (x_i - x_j) + \theta_{2y_j} (y_i - y_j) + \theta_{2z_j} (z_i - z_j).$$

Again, if this factor W_{2ji} is less than or equal to zero the function V_{ij} is set equal to zero; if not s_{ij}^2 is computed by the formula given previously and V_{ij} is computed by:



$$v_{ij} = (w_{1ij} : w_{2ji}) / (\left| \theta_{1x_i} \right| \cdot \left| \theta_{2z_j} \right| \cdot s_{ij}^4)$$

Or in expanded form it is:

$$v_{ij} = \frac{\left[\theta_{1x_i} (x_j - x_i) + \theta_{1y_i} (y_j - y_i) + \theta_{1z_i} (z_j - z_i) \right]}{\left| \theta_{1x_i} \right| \cdot \left| \theta_{2z_j} \right| \cdot \left[\theta_{2x_j} (x_i - x_j) + \theta_{2y_j} (y_i - y_j) + \theta_{2z_i} (z_i - z_j) \right] \cdot \left[(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2 \right]^2}$$

The sum of these integrands v_{ij} for all pairs of grid points P_i on A_1 and P_j on A_2 when multiplied by the standard projected area gives the desired approximation to the value of the integral. Similarly, if the values of dA_{1i} are summed over all grid points P_i on A_1 , the approximate value of the area of A_1 is obtained. Then $F_{1,2}$ can be obtained by dividing the value of the integral by π times the area of A_1 .

In the computer program, the summation proceeds by taking one point P_i on A_1 and summing over all points P_j on A_2 . Note that in practice, since the factors T_{1i} and T_{2j} both appear in numerator and denominator, they are cancelled and do not appear in the computations. Hence T_{1i} is computed only for the computation of the area A_1 or the differential shape factor from P_1 to A_2 . The factor T_{2j} is never computed in the shape factor computations, however since all the material is available, the program will compute the T_{2j} 's and the area A_2 on request. If the machine is instructed to compute the differential shape factor from point P_i on surface A_1 to surface A_2 , the general shape factor equations are used. The principal difference in the computation procedure is in the grid point routine where there is only the dependent variable of the one grid point P_i to compute. The single grid point P_i on A_1 of course reduces the shape factor integral to an integral over A_2 only, but this requires no additional computation since with only one grid point on A_1 there is no summing to be done.

If there is an interference test to be made, it would precede the computation of the factor W_{lij} . The test is accomplished by solving for the intersections of the line through P_i and P_j with the given surface which is designated as A_3 with quadric equation $\theta_3(x,y,z) = 0$. Then, if there is an intersection point which is both within the boundaries of A_3 and between P_i and P_j , the integrand V_{ij} is set equal to zero and the program advances to the next pair of grid points. If no such intersection point exists, the program makes the next required interference test or if all these tests are finished it proceeds to the computation of W_{lij} and eventually V_{ij} . The line containing P_i and P_j is defined by the three parametric equations:

$$x = x_i - (x_j - x_i) t = x_i + Lt$$

$$y = y_i - (y_j - y_i) t = y_i + Mt$$

$$z = z_i - (z_j - z_i) t = z_i + Nt$$

Substituting these values for x , y and z into $\theta_3(x,y,z) = 0$ gives:

$$\begin{aligned} & A_3 (x_i + Lt)^2 + B_3 (y_i + Mt)^2 + C_3 (z_i + Nt)^2 + 2D_3 (x_i + Lt)(y_i + Mt) \\ & + 2E_3 (x_i + Lt)(z_i + Nt) + 2F_3 (y_i + Mt)(z_i + Nt) + 2G_3 (x_i + Lt) \\ & + 2H_3 (y_i + Mt) + 2I_3 (z_i + Nt) + J_3 = 0 \end{aligned}$$

Clearing parentheses and collecting terms in powers of t gives the quadratic in t :

$$\begin{aligned} & \left[A_3 L^2 + B_3 M^2 + C_3 N^2 + 2 \left[L(D_3 M + E_3 N) + F_3 MN \right] \right] t^2 + 2 \left[L(A_3 x_i + D_3 y_i + \right. \\ & \left. E_3 z_i + G_3) + M(D_3 x_i + B_3 y_i + F_3 z_i + H_3) + N(E_3 x_i + F_3 y_i + C_3 z_i + I_3) \right] t + \\ & (A_3 x_i^2 + B_3 y_i^2 + C_3 z_i^2 + 2D_3 x_i y_i + 2E_3 x_i z_i + 2F_3 y_i z_i + 2G_3 x_i + 2H_3 y_i + \\ & 2I_3 z_i + J_3) = 0 \end{aligned}$$

Or this can be rewritten as

$$\{A_3 L^2 + B_3 M^2 + C_3 N^2 + 2[L(D_3 M + E_3 N) + F_3 MN]\} t^2 +$$

$$(L\theta_{3x_i} + M\theta_{3y_i} + N\theta_{3z_i}) t + \theta_3(x_i, y_i, z_i) = 0$$

This quadratic can then be solved, and for each real solution t such that $0 < t < 1$, the coordinates $x = x_i + Lt$, $y = y_i + Mt$ and $z = z_i + Nt$ are computed. Since $0 < t < 1$ insures that the point of intersection lies between P_i and P_j , it is only necessary to check with the boundaries of A_3 to complete the interference test.



APPENDIX C

PROGRAM ARRANGEMENT AND LISTING

The radiation configuration factor program is coded in FORTRAN IV and overlayed into two links. Figure C-1 shows the deck setup for use with the IBM 7094 computer. The program deck called "1568" in Link 0 is merely a control program for the flow to Link 1 or Link 2 during execution.

The subroutine program decks contained in Link 1 are:

1. Subroutine COMPILE reads the input for the surface and boundary conditions and computes the coefficients for the equation describing these surfaces and boundaries, stores the data on tape unit 11 and prints this input and the coefficients on the output tape.
2. Subroutine MDETR computes the value of determinants and these results are used in computing the coefficients for the general equation of a surface.

The subroutine program decks contained in Link 2 are:

1. Subroutine COMPUT reads the identification numbers of the surfaces for which shape factors are to be computed and prints the computed results, also stores the needed information for these computations on tape unit 8.
2. Subroutine VPANT searches tape unit 8 for the data of the surfaces for which shape factors are to be computed and brings this data into core storage.
3. Subroutine QMESS computes differential areas and performs a numerical integration to find the area of a given surface, also checks the input flag to see if any identification numbers of interfering surfaces are to be read from the input tape.
4. Subroutine GRID sets flags for later computation of grid points and dependent variables.
5. Subroutine POINT calculates the independent variables for the grid of the surfaces.

6. Subroutine DEPVAR calculates the dependent variables for the grid of the surfaces, and checks to see if all grid points lie within their specified boundaries.
7. Subroutine DIRECT computes the direction numbers for each point of the surfaces.
8. Subroutine FTEST checks the direction numbers for zero values.
9. Subroutine MTEST checks for shading of the surfaces if there were any interfering surfaces specified in the input.

Figure C-2 is a simplified flow diagram, showing the logical arrangement of the program and its two links. Table C-1 is a complete listing of the source program cards.



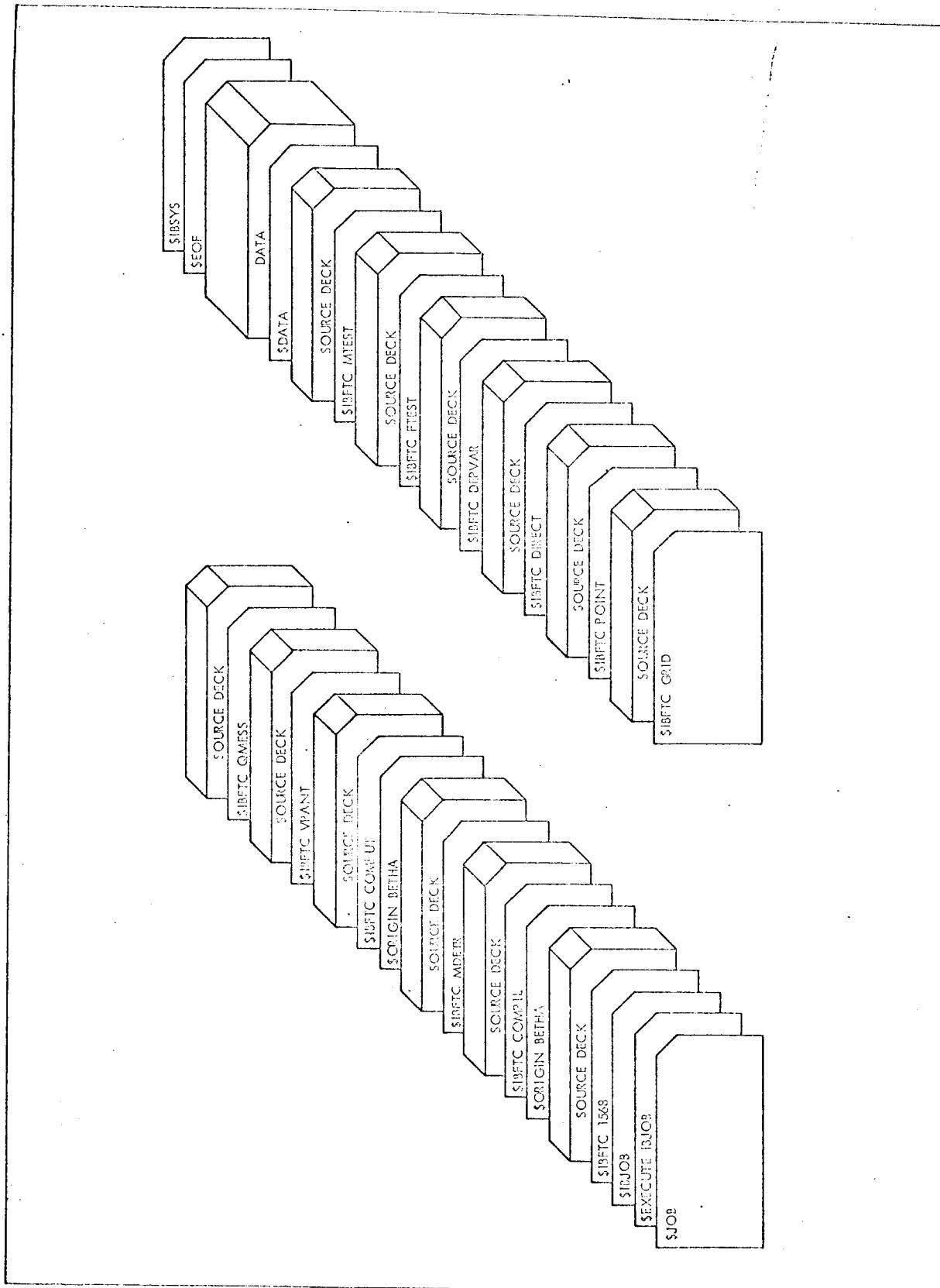


Figure C-1. FORTRAN IV Source Deck Setup for Radiation Configuration Factor Program

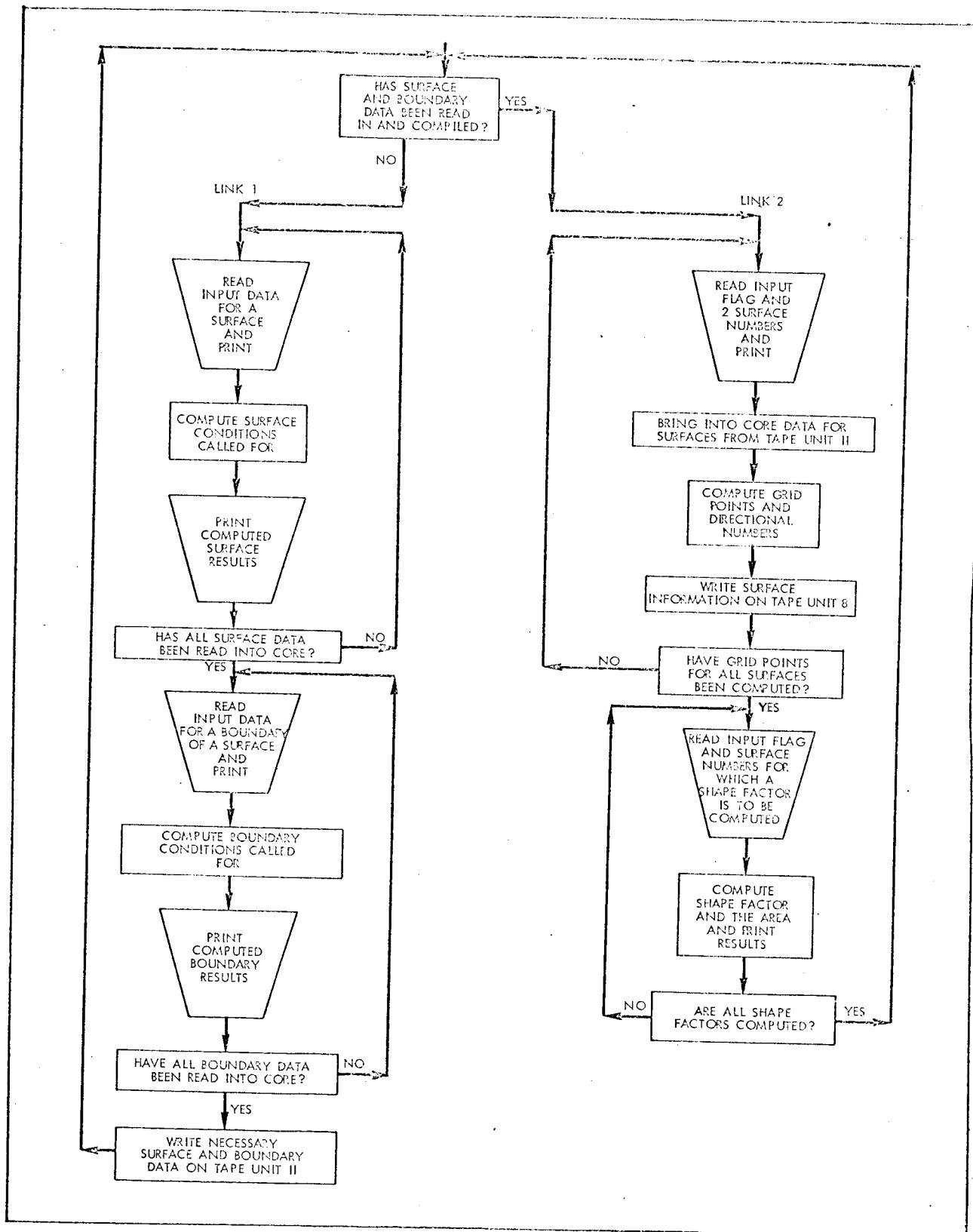


Figure C-2. Simplified Flow Diagram

TABLE C-1
PROGRAM LISTING

```

$IBFTC 1568 LIST,REF,DECK,M94,XR7
    REWIND 11                               1568001
    REWIND 8                               1568002
  2 CALL COMPIL                           1568003
    CALL COMPUT                           1568006
    REWIND 11                               1568007
    REWIND 8                               1568008
    GO TO 2
    END
$ORIGIN      RETHA
$IBFTC COMPIL LIST,REF,DECK,M94,XR7
    SUBROUTINE COMPIL
    DIMENSION DET(9,9),W(5),VAL(9)          15680020
    DIMENSION CA(100),CB(100),CC(100),CD(100),CE(100),CF(100),CG(100),15680030
    1 CH(100),CI(100),CJ(100),BA(100,6),BS(100,6),BC(100,6),BD(100,6), 15680040
    2 BE(100,6),BF(100,6),BG(100,6),BH(100,6),BI(100,6),BJ(100,6), 15680050
    3 X(10 ),Y(10 ),Z(10 ),XP(10 ),YP(10 ),ZP(10 ),DLP(10 ),DMP(10 ), 15680060
    4 DNP(10 ),LL(100),KNB(100),DSSI(100),CLIM1(100),CLIM2(100), 15680070
    5 QLIM3(100),QLIM4(100),GRIDS(100),DBSI(100,6),KOUNT(100) 15680080
  500 FORMAT (5I10)                         15680280
  501 FORMAT (7F10.3)                       15680290
  504 FORMAT (5I10,2F10.3)                  15680300
  510 FORMAT (5H J=I3,3X,29H SURFACE S(J) IS NOT DEFINED ) 15680310
  512 FORMAT (5H J=I3,3X, 3H K=I3,3X,31H BOUNDARY B(J,K) IS NOT DEFINED) 15680320
  1FD )
  514 FORMAT (46H FLAG IR IS GREATER THAN TWO IN MAIN PROGRAM ) 15680330
  520 FORMAT (1H1)                           15680340
  522 FORMAT (36H0   Q=(0,0,0) FOR SURFACE NO. I3 ) 15680350
  523 FORMAT (27H0   Q=(0,C,0) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680360
  524 FORMAT (36H0   Q=(1,0,0) FOR SURFACE NO. I3 ) 15680370
  525 FORMAT (27H0   Q=(1,C,0) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680380
  526 FORMAT (36H0   Q=(C,1,0) FOR SURFACE NO. I3 ) 15680390
  527 FORMAT (27H0   Q=(C,1,C) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680410
  528 FORMAT (36H0   Q=(0,0,1) FOR SURFACE NO. I3 ) 15680420
  529 FORMAT (27H0   Q=(0,0,1) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680430
  530 FORMAT (36H0   Q=(-1,0,0) FOR SURFACE NO. I3 ) 15680440
  531 FORMAT (27H0   Q=(-1,0,0) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680450
  532 FORMAT (36H0   Q=(0,-1,0) FOR SURFACE NO. I3 ) 15680460
  533 FORMAT (27H0   Q=(0,-1,C) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680470
  534 FORMAT (36H0   Q=(0,0,-1) FOR SURFACE NO. I3 ) 15680480
  535 FORMAT (27H0   Q=(0,0,-1) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680490
  536 FORMAT (36H0   Q=(1,1,0) FOR SURFACE NO. I3 ) 15680500
  537 FORMAT (27H0   Q=(1,1,C) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680510
  538 FORMAT (36H0   Q=(1,C,1) FOR SURFACE NO. I3 ) 15680520
  539 FORMAT (27H0   Q=(1,0,1) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680530
  540 FORMAT (36H0   Q=(0,1,1) FOR SURFACE NO. I3 ) 15680540
  541 FORMAT (27H0   Q=(0,1,1) FOR BDRY NO. I2,17H OF SURFACE NO. I3) 15680550
  545 FORMAT (35H0   COEFFICIENTS FOR SURFACE NO. I3,3X,2HA=E18.8,2X, 15680560
  12HB=E18.8 / 6X,2HC=E18.8,2X,2HD=E18.8,2X,2HE=E18.8,2X,2HF=E18.8 15680570
  2/6X,2HG=E18.8,2X,2HH=E18.8,2X,2HI=E18.8,2X,2HJ=E18.8 ) 15680580
  549 FORMAT (32H0   COEFFICIENTS FOR BDRY NO. I2,17H OF SURFACE NO. 15680590
  1I3,3X,2HA=E18.8,2X,2HB=E18.8 /6X,2HC=E18.8,2X,2HD=E18.8,2X,2HE=E18.8,2X,2HF= 15680600
  2.8,2X,2HG=E18.8 /6X,2HH=E18.8,2X,2HI=E18.8,2X,2HJ= 15680610
  3E18.8 ) 15680620
    KK=0                               15680630
    MAXS=0                           15680100
    WRITE (6,520)                      15680640
  .1 READ (5 ,500)IP,ISD,(LL(ISD),KNB(ISD)),ISDB 15680650
    WRITE (6,500) IP,ISD,LL(ISD),KNB(ISD),ISDB 15681001
    NN=0                               15680660
    IP=IP+2                           15680670

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TABLE C-1
(CONTINUED)

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2 IF (IP=10) 2,2,199
READ (5      ,501)DSSI(1SD),QLIM1(1SD),QLIM2(1SD),QLIM3(1SD), QLI15680680
1M4(1SD),GRIDS(1SD)                                         15680690
WRITE (6,501) DSSI(1SD),QLIM1(1SD),QLIM2(1SD),QLIM3(1SD),
1 QLIM4(1SD),GRIDS(1SD)                                         15680700
IR=0
MAXS=MAXS+1
KOUNT(MAXS)=1SD
GO TO (84,42,39,32,27,5,6,60,74,3),IP
15681003
15680710
15680101
15680111
15680720
15680730
15680740
15680750
15680760
15680770
15680780
15680790
15680800
15680810
15680820
15680830
15680840
15680850
15680860
15680870
15680880
15680890
15680900
15681004
15680910
15680920
15680930
15680940
15680950
15680960
15680970
15680980
15680990
15681000
15681010
15681020
15681030
15681040
15681050
15681060
15681070
15681080
15681090
15681100
15681110
15681120
15681130
15681140
15681150
15681160
15681170
15681180
15681190
15681200
15681210
15681220

C SET SURFACE COEFFS. = TO PREVIOUSLY CALCULATED COEFFS. - FLAG=8
C
3 A=CA(1SD)
B=CB(1SD)
C=CC(1SD)
D=CD(1SD)
E=CE(1SD)
F=CF(1SD)
G=CG(1SD)
H=CH(1SD)
QI=C1(1SD)
QJ=CJ(1SD)
GO TO 107
15680750
15680760
15680770
15680780
15680790
15680800
15680810
15680820
15680830
15680840
15680850
15680860
15680870
15680880
15680890
15680900
15680910
15680920
15680930
15680940
15680950
15680960
15680970
15680980
15680990
15681000
15681010
15681020
15681030
15681040
15681050
15681060
15681070
15681080
15681090
15681100
15681110
15681120
15681130
15681140
15681150
15681160
15681170
15681180
15681190
15681200
15681210
15681220

C READ IN SURFACE COEFICIENTS - FLAG=-1
C
84 READ (5      ,501)A,B,C,D,E,F,G,H,QI,QJ
WRITE (6,501) A,B,C,D,E,F,G,H,QI,QJ
D=D/2.0
E=E/2.0
F=F/2.0
G=G/2.0
H=H/2.0
QI=QI/2.0
QJ=QJ/2.0
GO TO 73
15680880
15680890
15680900
15681004
15680910
15680920
15680930
15680940
15680950
15680960
15680970
15680980
15680990
15681000
15681010
15681020
15681030
15681040
15681050
15681060
15681070
15681080
15681090
15681100
15681110
15681120
15681130
15681140
15681150
15681160
15681170
15681180
15681190
15681200
15681210
15681220

C READ IN THREE PLANE SURFACE POINTS - FLAG=0
C
42 READ (5      ,501)(X(I),Y(I),Z(I),I=1,3)
WRITE (6,501) (X(I),Y(I),Z(I),I=1,3)
MM=1
NN=3
A=0.0
B=0.0
C=0.0
D=0.0
E=0.0
F=0.0
DO 300 I=1,3
DET(I,1)=Y(I)
DET(I,2)=Z(I)
DET(I,3)=1.0
300 CONTINUE
CALL MDETR (3,ANS,DET)
G=ANS
DO 302 I=1,3
DET (I,1)=X(I)
DET (I,2)=Z(I)
DET (I,3)=-1.0
302 CONTINUE
CALL MDFTR (3,ANS,DET)
15681005
15681020
15681030
15681040
15681050
15681060
15681070
15681080
15681090
15681100
15681110
15681120
15681130
15681140
15681150
15681160
15681170
15681180
15681190
15681200
15681210
15681220

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TABLE C-1
(CONTINUED)

	H=ANS	15681230
	DO 304 I=1,3	15681240
	DET(I,1)=X(I)	15681250
	DET(I,2)=Y(I)	15681260
	DET(I,3)=1.0	15681270
304	CONTINUE	15681280
	CALL MDETR (3,ANS,DET)	15681290
	Q1=ANS	15681300
	QJ=-2.0*(G*X(1)+H*Y(1)+Q1*Z(1))	15681310
	GO TO 73	15681320
C	READ IN CENTER POINT AND RADIUS OF SPHERE - FLAG=1	15681330
C	39 READ (5 ,501)X(1),Y(1),Z(1),RADIUS	15681340
	WRITE (6,501) X(1),Y(1),Z(1),RADIUS	15681350
	A=1.0	15681360
	B=1.0	15681370
	C=1.0	15681380
	D=0.0	15681390
	E=0.0	15681400
	F=0.0	15681410
	G=-X(1)	15681420
	H=-Y(1)	15681430
	Q1=-Z(1)	15681440
	QJ= X(1)**2 +Y(1)**2+Z(1)**2-RADIUS**2	15681450
	GO TO 73	15681460
C	READ IN FOUR NON-COPLANAR SURFACE POINTS OF SPHERE - FLAG=2	15681470
C	32 READ (5 ,501)(X(1),Y(1),Z(1),I=1,4)	15681480
	WRITE (6,501) (X(I),Y(I),Z(I),I=1,4)	15681490
	MM=1	15681500
	NN=4	15681510
	A=1.0	15681520
	B=1.0	15681530
	C=1.0	15681540
	D=0.0	15681550
	E=0.0	15681560
	F=0.0	15681570
	G=0.0	15681580
	DO 306 I=1,4	15681590
	DET(I,1)=X(I)	15681600
	DET(I,2)=Y(I)	15681610
	DET(I,3)=Z(I)	15681620
	DET(I,4)=1.0	15681630
306	CONTINUE	15681640
	CALL MDETR (4,ANS,DET)	15681650
	QK=2.0*ANS	15681660
	DO 308 I=1,4	15681670
	W(I)=-1.0*(X(I)**2+Y(I)**2+Z(I)**2)	15681680
308	CONTINUE	15681690
	DO 310 I=1,4	15681700
	DET(I,1)=W(I)	15681710
	DET(I,2)=Y(I)	15681720
	DET(I,3)=Z(I)	15681730
	DET(I,4)=1.0	15681740
310	CONTINUE	15681750
	CALL MDETR (4,ANS,DET)	15681760
	G=ANS/QK	15681770
	DO 312 I=1,4	15681780
	DET(I,1)=X(I)	15681790
	DET(I,2)=W(I)	15681800
		15681810



TABLE C-1
(CONTINUED)

	DET(I,3)=Z(I)	15681820
	DET(I,4)=1.0	15681830
312	CONTINUE	15681840
	CALL MDETR (4,ANS,DET)	15681850
	H=ANS/GK	15681860
	DO 314 I=1,4	15681870
	DET(I,1)=X(I)	15681880
	DET(I,2)=Y(I)	15681890
	DET(I,3)=W(I)	15681900
	DET(I,4)=1.0	15681910
314	CONTINUE	15681920
	CALL MDETR (4,ANS,DET)	15681930
	QI=ANS/GK	15681940
	QJ= -(X(1)**2+Y(1)**2+Z(1)**2+2.0*G *X(1)+2.0*H*Y(1)+2.0*QI*Z(1))	15681950
	GO TO 73	15681960
C	READ IN 2-AXIS PTS AND RADIUS OF RT CIRCULAR CYL - FLAG=3	15681970
C	27 READ (5 ,501)(X(I),Y(I),Z(I),I=1,2),RADIUS	15681980
	WRITE (6,501) (X(I),Y(I),Z(I),I=1,2),RADIUS	15681990
	DL=X(2)-X(1)	15682000
	DM=Y(2)-Y(1)	15682010
	DN=Z(2)-Z(1)	15682020
	DD=SQRT(DL*DL+DM*DM+DN*DN)	15682030
	QLAMDA=DL/DD	15682040
	QMU=DM/DD	15682050
	QNEU=DN/DD	15682060
	A=0.0	15682070
	B=1.0	15682080
	G=0.0	15682100
	H=0.0	15682110
	QI=0.0	15682120
	QJ=-RADIUS**2	15682130
68	QK=A-B	15682140
	A=B+QK*QLAMDA**2	15682150
	C=B+QK*QNEU**2	15682160
	B=B+QK*QMU**2	15682170
	D=QK*QLAMDA*QMU	15682180
	E=QK*QLAMDA*QNEU	15682190
	F=QK*QMU*QNEU	15682200
71	QJ=QJ+A*X(1)**2+B*Y(1)**2+C*Z(1)**2 +2.0*((D*Y(1)+E*Z(1)-G)*X(1)+	15682210
1	F*Z(1)-H)*Y(1)-QI *Z(1))	15682220
72	G=G-A*X(1)-D*Y(1)-E *Z(1)	15682230
	H=H-D*X(1)-B*Y(1)-F *Z(1)	15682240
	QI=QI-E*X(1)-F*Y(1)-C *Z(1)	15682250
	GO TO 73	15682260
C	READ IN 3 SURFACE PTS AND DIRECTION NOS OF RT CIR CYL - FLAG=4	15682270
C	5 READ (5 ,501)(X(I),Y(I),Z(I),I=1,3),BL,BM,DN	15682280
	WRITE (6,501) (X(I),Y(I),Z(I),I=1,3),DL,DM,DN	15682290
	MM=1	15682300
	NN=3	15682310
	IR=-1	15682320
7	DD=SQRT(DL*DL+DM*DM+DN*DN)	15682330
	QLAMDA=DL/DD	15682340
	QMU=DM/DD	15682350
	QNEU=DN/DD	15682360
	IF (DN) 14,9,14	15682370
9	IF (DM) 12,10,12	15682380
10	DM1=Y(2)-Y(1)	15682390
		15682400



TABLE C-1
(CONTINUED)

	DN1=Z(2)-Z(1)	15682410
	DL1=0.0	15682420
	GO TO 16	15682430
12	DL1=X(2)-X(1)	15682440
	DN1=Z(2)-Z(1)	15682450
	DM1=-DL*DL1/DM1	15682460
	GO TO 16	15682470
14	DL1=X(2)-X(1)	15682480
	DM1=Y(2)-Y(1)	15682490
	DN1=-(DL*DL1+DM1*DM1)+DN1*DN1	15682500
16	DD1=SQRT(DL1*DL1+DM1*DM1+DN1*DN1)	15682510
	QLAM1=DL1/DD1	15682520
	QM1=DM1/DD1	15682530
	QNEU1=DN1/DD1	15682540
	QLAM2=QM1*QNEU1-QNEU1*QM1	15682550
	QM2=QNEU1*QLAM1-QLAM1*QNEU1	15682560
	QNEU2=QLAM1*QM1-QM1*QLAM1	15682570
	YP(1)=QLAM1*(X(2)-X(1))+QM1*(Y(2)-Y(1))+QNEU1*(Z(2)-Z(1))	15682580
	YP(2)=QLAM1*(X(3)-X(1))+QM1*(Y(3)-Y(1))+QNEU1*(Z(3)-Z(1))	15682590
	ZP(1)=QLAM2*(X(2)-X(1))+QM2*(Y(2)-Y(1))+QNEU2*(Z(2)-Z(1))	15682600
	ZP(2)=QLAM2*(X(3)-X(1))+QM2*(Y(3)-Y(1))+QNEU2*(Z(3)-Z(1))	15682610
	IF (IR) 22,48,19	15682620
19	IF (IR-2) 22,48,48	15682630
22	W(1)=-(YP(1)**2+ZP(1)**2)	15682640
	W(2)=-(YP(2)**2+ZP(2)**2)	15682650
	QK=2.0*(YP(1)*ZP(2)-YP(2)*ZP(1))	15682660
	HP=(W(1)*ZP(2)-ZP(1)*W(2))/QK	15682670
	OI=(YP(1)*W(2)-W(1)*YP(2))/QK	15682680
	G=HP*QLAM1+OI*QLAM2	15682690
	H=HP*QM1+OI*QM2	15682700
	QI=HP*QNEU1+OI*QNEU2	15682710
	A=0.0	15682720
	B=1.0	15682730
	CJ=0.0	15682740
	GO TO 68	15682750
48	YP(3)=QLAM1*(X(4)-X(1))+QM1*(Y(4)-Y(1))+QNEU1*(Z(4)-Z(1))	15682760
	YP(4)=QLAM1*(X(5)-X(1))+QM1*(Y(5)-Y(1))+QNEU1*(Z(5)-Z(1))	15682770
	ZP(3)=QLAM2*(X(4)-X(1))+QM2*(Y(4)-Y(1))+QNEU2*(Z(4)-Z(1))	15682780
	ZP(4)=QLAM2*(X(5)-X(1))+QM2*(Y(5)-Y(1))+QNEU2*(Z(5)-Z(1))	15682790
	DO 316 I=1,4	15682800
	DET(I,1)=ZP(:)**2	15682810
	DET(I,2)=YP(I)*ZP(I)	15682820
	DET(I,3)=YP(I)	15682830
	DET(I,4)=ZP(I)	15682840
316	CONTINUE	15682850
	CALL MDETR (4,ANS,DET)	15682860
	SP=2.0*ANS	15682870
	DO 318 I=1,4	15682880
	DET(I,1)=YP(I)**2	15682890
	DET(I,2)=YP(I)*ZP(I)	15682900
	DET(I,3)=YP(I)	15682910
	DET(I,4)=ZP(I)	15682920
318	CONTINUE	15682930
	CALL MDETR (4,ANS,DET)	15682940
	CP=-2.0*ANS	15682950
	DO 320 I=1,4	15682960
	DET(I,1)=YP(I)**2	15682970
	DET(I,2)=ZP(I)**2	15682980
	DET(I,3)=YP(I)	15682990
	DET(I,4)=ZP(I)	15683000
320	CONTINUE	15683010



TABLE C-1
(CONTINUED)

	CALL MDETR (4,ANS,DET)	15683020
	F=ANS	15683030
	DO 322 I=1,4	15683040
	DET(I,1)=YP(I)**2	15683050
	DET(I,2)=ZP(I)**2	15683060
	DET(I,3)=YP(I)*ZP(I)	15683070
	DET(I,4)=ZP(I)	15683080
322	CONTINUE	15683090
	CALL MDETR (4,ANS,DET)	15683100
	HP=-ANS	15683110
	DO 324 I=1,4	15683120
	DET(I,1)=YP(I)**2	15683130
	DET(I,2)=ZP(I)**2	15683140
	DET(I,3)=YP(I)*ZP(I)	15683150
	DET(I,4)=ZP(I)	15683160
324	CONTINUE	15683170
	CALL MDETR (4,ANS,DET)	15683180
	QI=ANS	15683190
	A=BP*QLAM1**2+CP*QLAM2**2+2.0*QLAM1*GLAM2*F	15683200
	B=BP*QMU1**2+CP*OMU2**2+2.0*QMU1*OMU2*F	15683210
	C=BP*CNEU1**2+CP*CNEU2**2+2.0*CNEU1*CNEU2*F	15683220
	D=BP*QLAM1*CMU1+CP*QLAM2*CMU2+(QLAM1*QMU2+OMU1*QLAM2)*F	15683230
	E=BP*QLAM1*QNEU1+CP*QLAM2*QNEU2+(QLAM1*QNEU2+QNEU1*QLAM2)*F	15683240
	F=BP*QMU1*QNEU1+CP*QMU2*QNEU2+(OMU1*QNEU2+QNEU1*QMU2)*F	15683250
	G=HP*QLAM1+QI*GLAM2	15683260
	H=HP*QMU1+QMU2*QI	15683270
	QI=HP*QNEU1+QNEU2*QI	15683280
	QJ=0.0	15683290
	GO TO 71	15683300
C	READ IN 5 PTS. AND DIRECTION NOS OF ARBITRARY CONIC CYL - FLAG=5	15683310
C	6 READ (5 ,501)(X(I),Y(I),Z(I),I=1,5),DL,DM,DN	15683320
	WRITE (6,501) (X(I),Y(I),Z(I),I=1,5),DL,DM,DN	15683330
	MM=1	15683340
	NN=5	15683350
	GO TO 7	15683360
C	READ IN 2 AXIS PTS P0 AND P4 + 3 PTS OF ARB CONIC OF REV -FLAG=6	15683370
C	60 READ (5 ,501)(X(I),Y(I),Z(I),I=1,5)	15683380
	WRITE (6,501) (X(I),Y(I),Z(I),I=1,5)	15683390
	MM=2	15683400
	NN=4	15683410
	DL=X(5)-X(1)	15683420
	DM=Y(5)-Y(1)	15683430
	DN=Z(5)-Z(1)	15683440
	DD=SQRT(DL*DL+DM*DM+DN*DN)	15683450
	QLAMDA=DL/DD	15683460
	QMU=DM/DD	15683470
	QNEU=DN/DD	15683480
	DLP(1)=X(2)-X(1)	15683490
	DLP(2)=X(3)-X(1)	15683500
	DLP(3)=X(4)-X(1)	15683510
	DMP(1)=Y(2)-Y(1)	15683520
	DMP(2)=Y(3)-Y(1)	15683530
	DMP(3)=Y(4)-Y(1)	15683540
	DNP(1)=Z(2)-Z(1)	15683550
	DNP(2)=Z(3)-Z(1)	15683560
	DNP(3)=Z(4)-Z(1)	15683570
	DO 61 I=1,3	15683580
		15683590
		15683600



TABLE C-1
(CONTINUED)

```

XP(I)=QLAMDA*DLP(I)+QMU*DMP(I)+QNEU*DNP(I) 15683610
YP(I)=(1.0-QLAMDA**2)*DLP(I)**2+(1.0-QMU**2)*DMP(I)**2+(1.0-QNEU**2)*DNP(I) 15683620
12 *DNP(I)**2-2.0*(QLAMDA*DLP(I)*(QMU*DMP(I)+QNEU*DNP(I))+ 15683630
2 QMU*QNEU*DNP(I)*DNP(I)) 15683640
61 CONTINUE 15683650
DO 62 I=1,3 15683660
DET(I,1)=YP(I) 15683670
DET(I,2)=XP(I) 15683680
DET(I,3)=1.0 15683690
62 CONTINUE 15683700
CALL MDETR (3,ANS,DET) 15683710
A=ANS 15683720
DO 63 I=1,3 15683730
DET(I,1)=XP(I)**2 15683740
DET(I,2)=XP(I) 15683750
DET(I,3)=-1.0 15683760
63 CONTINUE 15683770
CALL MDETR (3,ANS,DET) 15683780
B=ANS 15683790
DO 64 I=1,3 15683800
DET(I,1)=XP(I)**2 15683810
DET(I,2)=YP(I) 15683820
DET(I,3)=.5 15683830
64 CONTINUE 15683840
CALL MDETR (3,ANS,DET) 15683850
G=ANS 15683860
QJ=-A*XP(1)**2+B*YP(1)+2.0*XP(1)*G 15683870
QI=G*QNEU 15683880
H=G*QMU 15683890
G=G*QLAMDA 15683900
GO TO 68 15683910
C READ IN 9 SPACED SURFACE PTS OF AN ARB CONIC - FLAG=7 15683920
C 15683930
C 15683940
74 READ (5      ,501)(X(I),Y(I),Z(I),I=1,9) 15683950
WRITE (6,501) (X(I),Y(I),Z(I),I=1,9) 15681012
MM=1 15683960
NN=9 15683970
DO 76 I=1,9 15683980
DET(I,1)=Y(I)**2 15683990
DET(I,2)=Z(I)**2 15684000
DET(I,3)=X(I)*Y(I) 15684010
DET(I,4)=X(I)*Z(I) 15684020
DET(I,5)=Y(I)*Z(I) 15684030
DET(I,6)=X(I) 15684040
DET(I,7)=Y(I) 15684050
DET(I,8)=Z(I) 15684060
DET(I,9)=1.0 15684070
76 CONTINUE 15684080
CALL MDETR (9,ANS,DET) 15684090
A=ANS 15684100
DO 77 I=1,9 15684110
DET(I,1)=X(I)**2 15684120
DET(I,2)=Z(I)**2 15684130
DET(I,3)=X(I)*Y(I) 15684140
DET(I,4)=X(I)*Z(I) 15684150
DET(I,5)=Y(I)*Z(I) 15684160
DET(I,6)=X(I) 15684170
DET(I,7)=Y(I) 15684180
DET(I,8)=Z(I) 15684190
DET(I,9)=-1.0 15684200

```



TABLE C-1
(CONTINUED)

77	CONTINUE	
	CALL MDETR (9,ANS,DET)	15684210
	B=ANS	15684220
	DO 78 I=1,9	15684230
	DET(I,1)=X(I)**2	15684240
	DET(I,2)=Y(I)**2	15684250
	DET(I,3)=X(I)*Y(I)	15684260
	DET(I,4)=X(I)*Z(I)	15684270
	DET(I,5)=Y(I)*Z(I)	15684280
	DET(I,6)=X(I)	15684290
	DET(I,7)=Y(I)	15684300
	DET(I,8)=Z(I)	15684310
	DET(I,9)=1.0	15684320
	CONTINUE	15684330
	CALL MDETR (9,ANS,DET)	15684340
	C=ANS	15684350
	DO 79 I=1,9	15684360
	DET(I,1)=X(I)**2	15684370
	DET(I,2)=Y(I)**2	15684380
	DET(I,3)=Z(I)**2	15684390
	DET(I,4)=X(I)*Z(I)	15684400
	DET(I,5)=Y(I)*Z(I)	15684410
	DET(I,6)=X(I)	15684420
	DET(I,7)=Y(I)	15684430
	DET(I,8)=Z(I)	15684440
	DET(I,9)=-.5	15684450
79	CONTINUE	15684460
	CALL MDETR (9,ANS,DET)	15684470
	D=ANS	15684480
	DO 80 I=1,9	15684490
	DET(I,1)=X(I)**2	15684500
	DET(I,2)=Y(I)**2	15684510
	DET(I,3)=Z(I)**2	15684520
	DET(I,4)=X(I)*Y(I)	15684530
	DET(I,5)=Y(I)*Z(I)	15684540
	DET(I,6)=X(I)	15684550
	DET(I,7)=Y(I)	15684560
	DET(I,8)=Z(I)	15684570
	DET(I,9)=.5	15684580
80	CONTINUE	15684590
	CALL MDETR (9,ANS,DET)	15684600
	E=ANS	15684610
	DO 81 I=1,9	15684620
	DET(I,1)=X(I)**2	15684630
	DET(I,2)=Y(I)**2	15684640
	DET(I,3)=Z(I)**2	15684650
	DET(I,4)=X(I)*Y(I)	15684660
	DET(I,5)=X(I)*Z(I)	15684670
	DET(I,6)=X(I)	15684680
	DET(I,7)=Y(I)	15684690
	DET(I,8)=Z(I)	15684700
	DET(I,9)=-.5	15684710
81	CONTINUE	15684720
	CALL MDETR (9,ANS,DET)	15684730
	F=ANS	15684740
	DO 82 I=1,9	15684750
	DET(I,1)=X(I)**2	15684760
	DET(I,2)=Y(I)**2	15684770
	DET(I,3)=Z(I)**2	15684780
	DET(I,4)=X(I)*Y(I)	15684790
	DET(I,5)=X(I)*Z(I)	15684800
		15684810



TABLE C-1
(CONTINUED)

	DET(I,6)=Y(I)*Z(I)	15684820
	DET(I,7)=Y(I)	15684830
	DET(I,8)=Z(I)	15684840
	DET(I,9)=.5	15684850
82	CONTINUE	15684860
	CALL MDETR (9,ANS,DET)	15684870
	G=ANS	15684880
	DO 83 I=1,9	15684890
	DET(I,1)=X(I)**2	15684900
	DET(I,2)=Y(I)**2	15684910
	DET(I,3)=Z(I)**2	15684920
	DET(I,4)=X(I)*Y(I)	15684930
	DET(I,5)=X(I)*Z(I)	15684940
	DET(I,6)=Y(I)*Z(I)	15684950
	DET(I,7)=X(I)	15684960
	DET(I,8)=Z(I)	15684970
	DET(I,9)=-.5	15684980
83	CONTINUE	15684990
	CALL MDETR (9,ANS,DET)	15685000
	H=ANS	15685010
	DO 85 I=1,9	15685020
	DET(I,1)=X(I)**2	15685030
	DET(I,2)=Y(I)**2	15685040
	DET(I,3)=Z(I)**2	15685050
	DFT(I,4)=X(I)*Y(I)	15685060
	DET(I,5)=X(I)*Z(I)	15685070
	DET(I,6)=Y(I)*Z(I)	15685080
	DET(I,7)=X(I)	15685090
	DET(I,8)=Y(I)	15685100
	DET(I,9)=.5	15685110
85	CONTINUE	15685120
	CALL MDETR (9,ANS,DET)	15685130
	QI=ANS	15685140
	QJ=-(A*X(1)**2+B*Y(1)**2+C*Z(1)**2 +2.0*((D*Y(1)+E*Z(1)+G)*X(1)+	15685150
	1*(F*Z(1)+H)*Y(1)+O*I*Z(1)))	15685160
73	IF (QJ) 104,85,104	15685170
86	IF (A+2.0*G) 103,87,103	15685180
87	IF (B+2.0*H) 102,88,102	15685190
88	IF (C+2.0*QI) 101,89,101	15685200
89	IF (A) 100,90,100	15685210
90	IF (B) 99,91,99	15685220
91	IF (C) 98,92,98	15685230
92	IF (D) 97,93,97	15685240
93	IF (E) 96,94,96	15685250
94	IF (F) 95,108,95	15685260
95	ALPHA=F	15685270
	IF (KK) 420,420,421	15685280
420	WRITE (6,540) ISD	15685290
	GO TO 105	15685300
421	WRITE (6,541) KK,ISD	15685310
	GO TO 105	15685320
96	ALPHA=E	15685330
	IF (KK) 422,422,423	15685340
422	WRITE (6,538) ISD	15685350
	GO TO 105	15685360
423	WRITE (6,539) KK,ISD	15685370
	GO TO 105	15685380
97	ALPHA=D	15685390
	IF (KK) 424,424,425	15685400
424	WRITE (6,536) ISD	15685410

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TABLE C-1
(CONTINUED)

902	CONTINUE	15686030
	IF (NFLAG) 915,915,910	15686040
910	IF (KK) 906,906,908	15686050
906	WRITE (6,510)ISD	15686060
	GO TO 915	15686070
908	WRITE (6,512)KK,ISD	15686080
915	WRITE (6,950)(VAL(I),I=MM,NN)	15686090
950	FORMAT (6H 6E20.8/6X,3E20.8)	15686100
905	IF (IR-1) 107,213,213	15686110
107	CA(ISD)=A	15686120
	CB(ISD)=B	15686130
	CC(ISD)=C	15686140
	CD(ISD)=D	15686150
	CE(ISD)=E	15686160
	CF(ISD)=F	15686170
	CG(ISD)=G	15686180
	CH(ISD)=H	15686190
	CI(ISD)=QI	15686200
	CJ(ISD)=QJ	15686210
	D=2.0*E	15686220
	E=2.0*F	15686230
	F=2.0*E	15686240
	G=2.0*G	15686250
	H=2.0*H	15686260
	QI=2.0*QI	15686270
	WRITE (6,545)ISD,A,B,C,D,E,F,G,H,QI,QJ	15686280
	GO TO 1	15686290
108	IF (IR-1) 109,310,110	15686300
109	WRITE (6,510)ISD	15686310
	GO TO 1	15686320
110	WRITE (6,512)ISD,KK	15686330
	GO TO 200	15686340
C	STARTING BOUNDARY SPECIFICATIONS	15686350
C	199 WRITE (6,520)	15686360
200	READ (5 ,504)IQ,ISD,KK,ISDB,K2,(DBSI(ISD,KK))	15686370
	WRITE (6,504) IQ,ISD,KK,ISDB,K2,DBSI(ISD,KK)	15686380
	NN=0	15686390
	IQ=IC+2	15686400
	IF (IQ-8) 201,201,299	15686410
201	IR=1	15686420
	IF (KK-1) 770,202,204	15686430
202	WRITE (6,505)	15686440
505	FORMAT (1H0)	15686450
204	CONTINUE	15686460
	GO TO (206,207,42,27,210,211, 205,203),IQ	15686470
C	BDRY COEFFS. SET = TO PREVIOUS CALCULATED BDRY COEFFS. FLAG = 6	15686480
C	203 A=BA(ISDB,K2)	15686490
	B=BB(ISDB,K2)	15686500
	C=RC(ISDB,K2)	15686510
	D=BD(ISDB,K2)	15686520
	E=BE(ISDB,K2)	15686530
	F=BF(ISDB,K2)	15686540
	G=BG(ISDB,K2)	15686550
	H=BH(ISDB,K2)	15686560
	QI=BI(ISDB,K2)	15686570
	QJ=BJ(ISDB,K2)	15686580
	GO TO 213	15686590
		15686600
		15686610
		15686620



TABLE C-1
(CONTINUED)

	GO TO 105	15685420
425	WRITE (6,537)KK,ISD	15685430
	GO TO 105	15685440
98	ALPHA=C	15685450
	IF (KK) 426,426,427	15685460
426	WRITE (6,534)ISD	15685470
	GO TO 105	15685480
427	WRITE (6,535)KK,ISD	15685490
	GO TO 105	15685500
99	ALPHA=B	15685510
	IF (KK) 428,428,429	15685520
428	WRITE (6,532)ISD	15685530
	GO TO 105	15685540
429	WRITE (6,533)KK,ISD	15685550
	GO TO 105	15685560
100	ALPHA=A	15685570
	IF (KK) 430,430,431	15685580
430	WRITE (6,530)ISD	15685590
	GO TO 105	15685600
431	WRITE (6,531)KK,ISD	15685610
	GO TO 105	15685620
101	ALPHA=C+2.0*QI	15685630
	IF (KK) 432,432,433	15685640
432	WRITE (6,529)ISD	15685650
	GO TO 105	15685660
433	WRITE (6,529)KK,ISD	15685670
	GO TO 105	15685680
102	ALPHA=B+2.0*H	15685690
	IF (KK) 434,434,435	15685700
434	WRITE (6,526)ISD	15685710
	GO TO 105	15685720
435	WRITE (6,527)KR,ISD	15685730
	GO TO 105	15685740
103	ALPHA=A+2.0*G	15685750
	IF (KK) 436,436,437	15685760
436	WRITE (6,524)ISD	15685770
	GO TO 105	15685780
437	WRITE (6,525)KK,ISD	15685790
	GO TO 105	15685800
104	ALPHA=QJ	15685810
	IF (KK) 438,438,439	15685820
438	WRITE (6,522)ISD	15685830
	GO TO 105	15685840
439	WRITE (6,523)KK,ISD	15685850
105	A=A/ALPHA	15685860
	B=B/ALPHA	15685870
	C=C/ALPHA	15685880
	D=D/ALPHA	15685890
	E=E/ALPHA	15685900
	F=F/ALPHA	15685910
	G=G/ALPHA	15685920
	H=H/ALPHA	15685930
	QI=QI/ALPHA	15685940
	QJ=QJ/ALPHA	15685950
	IF (INN) 905,905,900	15685960
900	NFLAG=0	15685970
	DO 902 I=MM,NN	15685980
	VAL(I)=A*X(I)**2+B*Y(I)**2+C*Z(I)**2+2.0*D*X(I)*Y(I)+2.0*E*X(I)*Z(I)	15685990
	II)+2.0*F*Y(I)*Z(I)+2.0*G*X(I)+2.0*H*Y(I)+2.0*QI*Z(I)+QJ	15686000
	IF (ABS(VAL(I))-0.1) 902,904,904	15686010
904	NFLAG=2	15686020



TABLE C-1
(CONTINUED)

```

C      BDRY COEFFS. SET = TO CALCULATED SURFACE COEFFS. FLAG = -1    15686630
C      15686640
C      15686650
C      15686660
C      15686670
C      15686680
C      15686690
C      15686700
C      15686710
C      15686720
C      15686730
C      15686740
C      15686750
C      15686760
C      15686770
C      15686780
C      15686790
C      15686800
C      15681014
C      15686810
C      15686820
C      15686830
C      15686840
C      15686850
C      15686860
C      15686870
C      15686880
C      15686890
C      15686900
C      15686910
C      15681015
C      15686920
C      15686930
C      15686940
C      15686950
C      15686960
C      15686970
C      15686980
C      15686990
C      15687000
C      15687010
C      15687020
C      15687030
C      15687040
C      15687050
C      15687060
C      15687070
C      15687080
C      15681016
C      15687090
C      15687100
C      15687110
C      15687120
C      15687130
C      15687140
C      15687150
C      15687160
C      15687170
C      15687180
C      15687190
C      15687200

206 A=CA(ISDB)                                15686630
      B=CB(ISDB)                                15686640
      C=CC(ISDB)                                15686650
      D=CD(ISDB)                                15686660
      E=CE(ISDB)                                15686670
      F=CF(ISDB)                                15686680
      G=CG(ISDB)                                15686690
      H=CH(ISDB)                                15686700
      QI=CI(ISDB)                                15686710
      QJ=CJ(ISDB)                                15686720
      GO TO 213                                 15686730
      15686740
      15686750
      15686760

C      READ IN BOUNDARY COEFICIENTS FLAG=5      15686770
C      15686780
C      15686790

205 READ (5 ,501)A,B,C,D,E,F,G,H,QI,QJ      15686800
      WRITE (6,501) A,B,C,D,E,F,G,H,QI,QJ
      D=D/2.0                                     15681014
      E=E/2.0                                     15686810
      F=F/2.0                                     15686820
      G=G/2.0                                     15686830
      H=H/2.0                                     15686840
      QI=QI/2.0                                   15686850
      GO TO 73                                    15686860
      15686870

C      READ IN TWO BOUNDARY POINTS FLAG=0        15686880
C      15686890
C      15686900

207 READ (5 ,501)(X(I),Y(I),Z(I),I=1,2)      15686910
      WRITE (6,501) (X(I),Y(I),Z(I),I=1,2)
      MM=1                                         15681015
      NN=2                                         15686920
      DL=X(2)-X(1)                                15686930
      DM=Y(2)-Y(1)                                15686940
      DN=Z(2)-Z(1)                                15686950
      G=DN*CH(ISD)-DM*CI(ISD)                      15686960
      H=DL*CI(ISD)-DN*CG(ISD)                      15686970
      QI=DM*CG(ISD)-DL*CH(ISD)                      15686980
      QJ=-2.0*(G*X(1)+H*Y(1)+QI*Z(1))           15686990
      A=0.0                                         15687000
      B=0.0                                         15687010
      C=0.0                                         15687020
      D=0.0                                         15687030
      E=0.0                                         15687040
      F=0.0                                         15687050
      GO TO 73                                    15687060
      15687070
      15687080
      15681016
      15687090
      15687100
      15687110
      15687120
      15687130
      15687140
      15687150
      15687160
      15687170
      15687180
      15687190
      15687200

210 READ (5 ,501)(X(I),Y(I),Z(I),I=1,3),DL,DM,DN   15687100
      WRITE (6,501) (X(I),Y(I),Z(I),I=1,3),DL,DM,DN
      MM=1                                         15687110
      NN=3                                         15687120
      GO TO 7                                     15687130
      15687140
      15687150
      15687160
      15687170
      15687180
      15687190
      15687200

211 IR=2                                       15687100
      GO TO 6                                     15687110
      15687120
      15687130
      15687140
      15687150
      15687160
      15687170
      15687180
      15687190
      15687200

213 BA(ISD,KK)=A                             15687100
      BB(ISD,KK)=B                             15687110
      BC(ISD,KK)=C                             15687120
      BD(ISD,KK)=D                             15687130
      BE(ISD,KK)=E                             15687140
      BF(ISD,KK)=F                             15687150
      BG(ISD,KK)=G                             15687160
      15687170
      15687180
      15687190
      15687200
  
```



TABLE C-1
(CONTINUED)

BH(ISD,KK)=H	15687210
BI(ISD,KK)=QI	15687220
BJ(ISD,KK)=QJ	15687230
D=2.0*D	15687240
E=2.0*E	15687250
F=2.0*F	15687260
G=2.0*G	15687270
H=2.0*H	15687280
QI=2.0*QI	15687290
WRITE (6,549)KK,ISD,A,B,C,D,E,F,G,H,QI,QJ	15687300
IF (IR-2) 200,200,216	15687310
216 WRITE (6,514)	15687320
GO TO 770	15687330
299 WRITE (6,520)	15687340
IF (MAXS.GT.0) GO TO 6077	COMPL01
7000 RETURN	COMPL02
6077 IF (IQ.LT.23) GO TO 7001	15685001
6098 READ (11) IDD	15685002
IF (IDD) 6099,6099,6098	15685003
6099 BACKSPACE 11	15685004
7001 DO 7005 J=1,MAXS	COMPL03
I=KOUNT(J)	15680507
NB=KNB(I)	COMPL04
IF (NB.GT.0.AND.NB.LT.7) GO TO 7003	COMPL 5
NB=1	COMPL06
7003 WRITE (11) I,CA(I),CB(I),CC(I),CD(I),CE(I),CF(I),CG(I),CH(I),CI(I)	
1 ,CJ(I),LL(I),KNB(I),NB,DSSI(I),QLIM1(I),QLIM2(I),QLIM3(I),	
2 QLIM4(I),GRIDS(I),(BA(I,K),BB(I,K),BC(I,K),BD(I,K),BE(I,K),BF(I,K	
3),BG(I,K),BH(I,K),BI(I,K),BJ(I,K),CRSI(I,K),K=1,NB)	
7005 CONTINUE	COMPL100
IDD=0	COMPL102
WRITE (11) IDD	COMPL103
END FILE 11	1568004
REWIND 11	1568005
GO TO 7000	15685001
770 CALL EXIT	15687380
STOP	15687390
END	15687400
\$IBFTC MDETR LIST,REF,DECK,M94,XR7	
SUBROUTINE MDETR (N,C,A)	15680020
DIMENSION A(9,9),B(10)	15680030
LN=N-1	15680050
DO 18 I=1,LN	15680060
MI=I+1	15680070
DO 18 K=MI,N	15680080
IF (A(I,I)) 19,3,19	15680090
3 K1=I+1	15680100
DO 6 K2=K1,N	15680110
IF (A(I,K2)) 4,6,4	15680120
4 DO 5 JJ=1,N	15680130
B(JJ)=A(JJ,K2)	15680140
A(JJ,K2)=A(JJ,I)	15680150
A(JJ,I)=-B(JJ)	15680160
5 CONTINUE	15680170
GO TO 19	15680180
6 CONTINUE	15680190
C=0.0	15680200
GO TO 100	15680210
19 AA=A(K,I)/A(I,I)	15680220
DO 18 J=I,N	15680230
A(K,J)=A(K,J)-AA*A(I,J)	15680240



TABLE C-1
(CONTINUED)

```

18  CONTINUE          15680250
    B(1)=1.0          15680260
    DO 32 L=1,N       15680270
    B(L+1)=B(L)*A(L,L) 15680280
32  CONTINUE          15680290
    C=B(N+1)          15680300
100 RETURN            15680310
    END               15680320
$ORIGIN      BETHA
$IBFTC COMPUT LIST,REF,DECK,M94,XR7
    SUBROUTINE COMPUT          15680020
    DIMENSION CA(10 ),CB(10 ),CC(10 ),CD(10 ),CE(10 ),CF(10 ),CG(10 ),15680030
1   CH(10 ),CI(10 ),CJ(10 ),BA(10 ,6),BB(10 ,6),BC(10 ,6),BD(10 ,6), 15680040
2   BE(10 ,6),BF(10 ,6),BG(10 ,6),BH(10 ,6),BI(10 ,6),BJ(10 ,6), 15680050
3   X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),DMP(875), 15680060
4   DNP(875),LL(10 ),KNB(10 ),DSSI(10 ),QLIM1(10 ),QLIM2(10 ), 15680070
5   QLIM3(10 ),QLIM4(10 ),GRIDS(10 ),DBSI(10 ,6)          15680080
    DIMENSION DKP(875),DL(875),DM(875),DN(875),DK(875),PHI(875), 15680090
1   PSI(875),MIS(10 ),T(875),U(875),V(875),QQ(875), VPST(875) 15680100
    COMMON /ECCM/ CA, CB, CC, CD, CE, CF          15680130
1   , CG, CH, CI, CJ, BA, BB          15680140
2   , BC, BD, BE, BF, BG, BH          15680150
3   , BI, BJ, X, Y, Z, XP          15680160
4   , YP, ZP, DLP, DMP, DNP, LL          15680170
5   , KNB, DSSI, QLIM1, QLIM2, QLIM3, QLIM4          15680180
6   , GRIDS, DBSI, DKP, DL, DM, DN          15680190
7   , DK, PHI, PSI, MIS, T, U          15680200
8   , V, QQ, VPST, NGO, NDO, NP          15680210
9   , IP, IL, IH, IT, IS, NC          15680220
1   , NLD, IU, NFT, NSIGMA, NPP, KP          15680230
2   , A, B, C, D, E, F          15680240
3   , G, H, OI, QJ, RIGHT, DEL1          15680250
4   , DEL2, DEL1P, DEL2P, NBDD, NVP,NOIS,NAREA,LITEMS 15680260
    LITEMS=0          COMPU001
    IH=0          COMPU002
    NFLAG=1          COMPU003
2   READ (5,200) KNTRL,IT,IS,NOIS,NAREA          COMPU004
    WRITE (6,200) KNTRL,IT,IS,NOIS,NAREA          15681017
    NBDD=0          COMPU005
    IF (KNTRL) 3,5,4          COMPU006
3   IDD=0          COMPU007
    WRITE (8) IDD          COMPU008
    REWIND 8          COMPU009
    GO TO 5          COMPU010
4   WRITE (6,210)          COMPU011
    GO TO (402,401),NFLAG          COMPU012
401  IDD=0          COMPU013
    WRITE (8) IDD          COMPU014
    END FILE 8          COMPU015
    REWIND 8          COMPU016
402  NFLAG=1          COMPU017
    GO TO (101,104,106,770),KNTRL          COMPU018
5   GO TO (6,9),NFLAG          COMPU019
6   NFLAG=2          COMPU020
7   READ(8) IDD          COMPU021
    IF (IDD) 410,410,7          COMPU022
410  BACKSPACE 8          COMPU023
9   LITEMS=IT          COMPU026
    IT=1          COMPU027
    KFLG=1          COMPU028
    GO TO 415          COMPU029

```



TABLE C-1
(CONTINUED)

411	IF (IS.LT.1) GO TO 2	
412	LTEMS=IS	COMPU030
	IS=1	COMPU033
	KFLG=?	COMPU034
	IT=1	COMPU03K
415	READ (11) IDD,CA(1),CB(1),CC(1),CD(1),CE(1),CF(1),CG(1),CH(1), 1 CI(1),CJ(1),LL(1),KNB(1),NB,DSSI(1),QLIM1(1),QLIM2(1),QLIM3(1), 2 QLIM4(1),GRIDS(1),(BA(1,K),BB(1,K),BC(1,K),BD(1,K),BE(1,K), 3 BF(1,K),BG(1,K),BH(1,K),BI(1,K),BJ(1,K),DBSI(1,K),K=1,NB) IF (IDD.NE.LTEMS) GO TO 415	COMPU026 COMPU029 COMPU030 COMPU040 COMPU041 COMPU042 COMPU025 COMPU043 COMPU044 COMPU045 COMPU046 COMPU047 15680600 15680610 15680620 15680630 15680640 1568 65 15680104 15680105 15680106 15680660 15680680 15680121 15680122 15680126 15680127 15680690 15680700 15680710 15680720 15680730 15680740 15680750 15680760 15680770 15680780 15680131 15680132 15681110 15681120 15681130 15680135 15680136 1568 4 15681018 15681150 15681160 15681170 15681180 15681190 15681200 15681210 15681220 15681230 15681240 15681250
416	NGO=1	
	GO TO 418	
417	NGO=4	
418	CALL GRID	
	IF (RIGHT-1.0) 10,113,11	
10	WRITE (6,201) LTEMS	15680600
	GO TO (411,2),KFLG	15680610
11	WRITE (6,202)	15680620
	GO TO 770	15680630
113	GO TO (419,420),KFLG	15680640
419	NLD=1	1568 65
	GO TO 421	15680104
420	NLD=2	15680105
421	CALL DIRECT	15680660
	IF (NP) 425,425,330	15680680
425	WRITE (6,8000) LTEMS	15680121
8000	FORMAT (18H0 SURFACE NUMBER I4,22H WAS NOT PUT ON TAPE)	15680122
8001	FORMAT (18H0 SURFACE NUMBER I4,18H WAS PUT ON TAPE)	15680126
	GO TO (411,2),KFLG	15680127
330	IF (KNS(IT)) 310,310,312	15680690
310	NB=1	15680700
	DBSI(IT,1)=0.0	15680710
	GO TO 314	15680720
312	NB=KNB(IT)	15680730
314	WRITE (8) LTEMS,NP,NB,DEL1,DEL2,CA(IT),CB(IT),CC(IT),CD(IT),CE(IT), 1,CF(IT),CG(IT),CH(IT),CI(IT),CJ(IT),(BA(IT,I),BB(IT,I),BC(IT,I), 2 BD(IT,I),BE(IT,I),BF(IT,I),BG(IT,I),BH(IT,I),BI(IT,I),BJ(IT,I), 3 DBSI(IT,I),I=1,NB),(X(I),Y(I),Z(I),DL(I),DM(I),DN(I),DK(I),I=1,NP)	15680740 15680750 15680760 15680770 15680780 15680131 15680132 15681110 15681120 15681130 15680135 15680136 1568 4 15681018 15681150 15681160 15681170 15681180 15681190 15681200 15681210 15681220 15681230 15681240 15681250
101	IH=0	
	GO TO 51	
104	IH=-1	
	LTEMS=IT	
	IT=1	
	READ (5,204) LL(IT),NP,DSSI(IT),(X(I),Y(I),I=1,NP)	
	WRITE (6,204) LL(IT),NP,DSSI(IT),(X(I),Y(I),I=1,NP)	
	NBDD=2	
	MDEP=LL(IT)	
	GO TO (30,32,35),MDEP	
30	DO 31 I=1,NP	15681170
	Z(I)=Y(I)	15681180
	Y(I)=X(I)	15681190
31	CONTINUE	15681200
	GO TO 35	15681210
32	DO 33 I=1,NP	15681220
	Z(I)=Y(I)	15681230
33	CONTINUE	15681240
		15681250



TABLE C-1
(CONTINUED)

35	NGO=2	15681260
	REWIND 11	15681270
37	READ (11) IDD,CA(1),CB(1),CC(1),CD(1),CE(1),CF(1),CG(1),CH(1), 1 CI(1),CJ(1),LL(3),KNB(1),NB,DBSI(3),QLIM1(1),QLIM2(1),QLIM3(1), 2 QLIM4(1),GRIDS(1),(BA(1,K),BD(1,K),BC(1,K),BD(1,K),BE(1,K), 3 BF(1,K),BG(1,K),BH(1,K),BI(1,K),BJ(1,K),DBSI(1,K),K=1,NB) IF (IDD.NE.LTEMS) GO TO 37	COMPUZ48 COMPU049 COMPU050 COMPU051 COMPU052 COMPU053 15681370
	IF (KNB(1).GT.0) GO TO 82	15681380
81	KNB(IT)=0 GO TO 39	15681390
82	KNB(IT)=NB	15681400
39	CALL GRID NLD=1 CALL DIRECT IT=LTEMS IF (NP) 38,38,50	15681410 15681420 15680141 15681430 15680142
38	WRITE (6,8003) LTEMPS	15680143
8003	FORMAT (23HO NO POINTS ON SURFACE I4) GO TO 42	15680145
106	IH=1 GO TO 119	15681440
50	NVP=2 CALL VPANT IF (NVP) 42,42,2	15681450 15681460 15681470 15681480
42	IF (NOIS) 2,2,360	15681500
360	READ (5 ,271)(MIS(I),I=1,NOIS) WRITE (6,271) (MIS(I),I=1,NOIS)	15681510 15681019
271	FORMAT (7I10) GO TO 2	15681520 15681530
51	NVP=1 CALL VPANT IF (NVP) 42,42,44	15681540 15681550 15681560
44	DELSAT=DEL1*DEL2 SUM=0.0 DO 53 I=1,NP. SUM=SUM+VPST(I)	15681570 15681580 15681590 15681600
53	CONTINUE WRITE (6,261)DEL1,DEL2,DEL1P,DEL2P	15681610 15681620
261	FORMAT (9H DELS 4E20.8) VSST=DELSAT*SUM WRITE (6,205)IT,IS,VSST GO TO 120	15681630 15681640 15681650 15681660 15681670
119	IS=IT REWIND 8	15681680
56	READ (8)IDD IF (IDD) 60,60,54	15681690 15681700
54	IF (IT-IDD) 56,61,56	15681710
60	WRITE (6,201)IT GO TO 2	15681720 15681730
61	BACKSPACE 8 READ (8)IT,NP,NB,DEL1,DEL2,A,B,C,D,E,F,G,H,QI,QJ,(BA(1,I), 1 BB(1,I),BC(1,I),BD(1,I),BE(1,I),BF(1,I),BG(1,I),BH(1,I),BI(1,I), 2 BJ(1,I),DBSI(1,I),I=1,NB),(X(I),Y(I),Z(I),DL(I),DM(I),DN(I), 3 DK(I),I=1,NP)	15681750 15681760 15681770 15681780
120	DELSAT=DEL1*DEL2 SUM=0.0 DO 62 I=1,NP SUM=SUM+SQRT(DL(I)**2+DM(I)**2+DN(I)**2)/DK(I)	15681790 15681800 15681810 15681820
62	CONTINUE SAT=DELSAT*SUM WRITE (6,206)IT,SAT	15681830 15681840 15681850



TABLE C-1
(CONTINUED)

```

222 IF (IH) 2,222,2
      VFST=VGST/SAT
      WRITE (6,208) IT,IS,VFST
      GO TO 2
200 FORMAT (5I10)
201 FORMAT (16H0 SURFACE NO. I3,50H WAS NOT PUT ON TAPE,NO DEP. PTS15681910
1 WERE CALCULATED ) 15681860
15681870
15681880
15681890
15681900
15681920
15681930
15681940
15681950
15681960
15681970
15681980
15681990
15682000
15682010
202 FORMAT (62H0 RIGHT,A FLAG SET IN GRID IS GREATER THAN ONE WHA 15681920
1HOPPE )
204 FORMAT (2I10,1F10.3/(7F10.3)) 15681940
205 FORMAT ( 6H0 S=I3,2X,4H T=I3,2X,14H A(S)F(S,T)= E20.8) 15681950
208 FORMAT ( 6H0 S=I3,2X,4H T=I3,2X,11H F(S,T)= E20.8 ) 15681970
206 FORMAT ( 6H0 S=I3,2X,12H AREA (S)= E20.8) 15681980
210 FORMAT (1H0/1H0 )
770 RETURN;
END
$IBFTC VPANT LIST,REF,DECK,M94,XR7
SUBROUTINE VPANT
DIMENSION CA(10 ),CB(10 ),CC(10 ),CD(10 ),CE(10 ),CF(10 ),CG(10 ),15680020
1 CH(10 ),CI(10 ),CJ(10 ),SA(10 ,6),SB(10 ,6),BC(10 ,6),SD(10 ,6),15680030
2 BE(10 ,6),BF(10 ,6),BG(10 ,6),BH(10 ,6),BI(10 ,6),BJ(10 ,6),15680040
3 X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),DMP(875),15680050
4 DNP(875),LL(10 ),KNB(10 ),DSSI(10 ),QLIM1(10 ),QLIM2(10 ),15680060
5 QLIM3(10 ),QLIM4(10 ),GRIDS(10 ),DRSI(10 ,6) 15680070
DIMENSION DKP(875),DL(875),D1(875),DN(875),DK(875),PHI(875),15680080
1 PSI(875),MIS(10 ),T(875),J(875),V(875),QQ(875),VPST(875) 15680090
15680100
COMMON /ECOM/ CA, CB, CC, CD, CE, CF,15680130
1 CG, CH, CI, CJ, BA, BB,15680140
2 BC, BD, BE, BF, BG, BH,15680150
3 BI, BJ, X, Y, Z, XP,15680160
4 YP, ZP, DLP, DMR, DNP, LL,15680170
5 KNB, DSSI, QLIM1, QLIM2, QLIM3, QLIM4,15680180
6 GRIDS, DRSI, DKP, DL, DM, DN,15680190
7 DK, PHI, PSI, MIS, T, U,15680200
8 V, QQ, VPST, NGO, NDO, NP,15680210
9 IP, IL, IH, IT, IS, NC,15680220
1 NLD, IU, MFT, NSIGMA, NPP, KP,15680230
2 A, B, C, D, E, F,15680240
3 G, H, CI, QJ, RIGHT, DEL1,15680250
4 DEL2, DELIP, DEL2P, NRD0, NVP,NQIS,NAREA,LTEMS,15680260
REWIND 8
KCUNT=0
GO TO 1,5),NVP
1 READ (8 )IDD
IF (IDD) 17,17,11 15680480
11 IF (IT-IDD) 12,13,12 15680490
12 IF (IS-IDD) 1,14,1 15680500
13 KOUNT=KOUNT+1 15680510
BACKSPACE 8 15680520
READ (8 )IT,NP,NB,DEL1,DEL2,A,B,C,D,E,F,G,H,QI,QJ,(BA(1,I),15680530
1 BB(1,I),BC(1,I),BD(1,I),BE(1,I),BF(1,I),BG(1,I),BH(1,I),BI(1,I),15680540
2 BJ(1,I),DBSI(1,I),I=1,NB),(X(I),Y(I),Z(I),DL(I),DM(I),DN(I),15680550
3 DK(I),I=1,NP) 15680560
IF (IT-IS) 8,14,8 15680570
8 IF (KOUNT-2) 1,6,18 15680580
14 KOUNT=KOUNT+1 15680600
BACKSPACE 8 15680610
READ (8 )IS,NPP,NB,DEL1P,DEL2P,A,B,C,D,E,F,G,H,QI,QJ,(BA(1,I)15680620
1, BB(1,I),BC(1,I),BD(1,I),BE(1,I),BF(1,I),BG(1,I),BH(1,I),BI(1,I),15680630
2 BJ(1,I),DBSI(1,I),I=1,NB),(XP(I),YP(I),ZP(I),DLP(I),DMP(I),15680640
3 DNP(I),DKP(I),I=1,NPP) 15680650

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TABLE C-1
(CONTINUED)

```

IF (KOUNT=2) 1,6,18
5 KOUNT=1
2 READ (8 )IDD
IF (IDD) 17,17,3
3 IF (IS=IDD) 2,4,2
4 BACKSPACE 8
READ (8 )IS,NPP,NB,DEL1P,DFL2P,A,B,C,D,E,F,G,H,QI,QJ,(BA(1,I))15680720
1, BB(1,I),BC(1,I),BD(1,I),BE(1,I),SF(1,I),BG(1,I),BH(1,I),BI(1,I),15680730
2 BJ(1,I),DSSI(1,I),I=1,NB),(XP(I),YP(I),ZP(I),DLP(I),DMP(I),15680740
3 DNP(I),DKP(I),I=1,NPP)15680750
6 CALL QMESS
IF (IH) 9,25,19
9 DO 21 I=1,NP
PIFST=VPST(I)*DK(I)/SQRT(DL(I)**2+DM(I)**2+DN(I)**2)
WRITE (6,50)I,IT,IS,PIFST,X(I),Y(I),Z(I)15680760
21 CONTINUE
GO TO 25
15680770
15680780
15680790
15680795
15680800
15680810
15680820
15680830
15680840
15680850
15680860
15680870
15680880
15680890
15680895
15680900
15680910
15680920
15680930
15680940
15680950
15680960
$IBFTC QMESS LIST,REF,DECK,M94,XR7
SUBROUTINE QMESS
DIMENSION CA(10 ),CB(10 ),CC(10 ),CD(10 ),CE(10 ),CF(10 ),CG(10 ),15680020
1 CH(10 ),CI(10 ),CJ(10 ),BA(10 ,6),BB(10 ,6),BC(10 ,6),BD(10 ,6),15680030
2 BE(10 ,6),BF(10 ,6),BG(10 ,6),BH(10 ,6),BI(10 ,6),BJ(10 ,6),15680040
3 X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),DMP(875),15680050
4 DNP(875),LL(10 ),KNB(10 ),DSSI(10 ),QLIM1(10 ),QLIM2(10 ),15680060
5 QLIM3(10 ),QLIM4(10 ),GRIDS(10 ),DSSI(10 ,6)15680070
DIMENSION DKP(875),DL(875),DM(875),DN(875),OK(875),PHI(875),15680080
1 PSI(875),VIS(10 ),T(875),U(875),V(875),QG(875),VPST(875)15680100
COMMON /ECOM/ CA, CB, CC, CD, CE, CF15680130
1 , CG , CH , CI , CJ , RA , BB15680140
2 , BC , BD , BE , BF , BG , RH15680150
3 , BI , BJ , X , Y , Z , XP15680160
4 , YP , ZP , DLP , DMP , DNP , LL15680170
5 , KNB , DSSI , QLIM1 , QLIM2 , QLIM3 , QLIM415680180
6 , GRIDS , DRSI , DKP , DL , DM , DN15680190
7 , DK , PHI , PSI , VTS , T , U15680200
8 , V , QG , VPST , NGO , NDO , NP15680210
9 , IP , IL , IH , IT , IS , NC15680220
1 , NLD , IU , NFT , NSIGMA , NPP , KP15680230
2 , A , B , C , D , E , F15680240
3 , G , H , QI , QJ , RIGHT , DEL115680250
4 , DEL2 , DEL1P , DFL2P , NRDD , NVP , NOIS , NFLAG , LITEMS15680260
200 FORMAT (7I10)15680450
201 FFORMAT (62HO) NO INTERFERING SURFACES APPEAR ON TAPE FOR SURFAC15680460
1E NOS. I3, 6H AND I3 )15680470
202 FORMAT (33HO) NO. OF INTERFERING SURFACES= I3,3X,39H NO. OF INTER15680480
1FERING SURFACES ASKED FOR= I3,3X,21H BETWEEN SURFACE NOS. I3,3X,15680490
2 4HAND I3 )15680500

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TABLE C-1
(CONTINUED)

```

206 FORMAT (6H    VA= E20.8 /5H    T=I2,2X,11H AREA (T)= E20.8)      15680510
210 FORMAT (5H    S= I3,3X,2HIT=I3,5X,4CHF(S,T) WITH INTERFERENCE BY SUR15680520
1FACE NCS. 1015 )
REWIND 11
KLM=0
KP=0
VA=0.0
PI=3.14159265
DELSAS=DEL1P*DEL2P
IF (NOIS) 6,6,18
18 READ (5      ,200)(MIS(I),I=1,NOIS)
WRITE (6,210)IT,IS,(MIS(I),I=1,NOIS)
GO TO 6
4 SUM=0.0
DO 7 J=1,NPP
SUM=SUM+QQ(J)/DKP(J)
7 CONTINUE
VPST(KP)=DELSAS*SUN/(PI*DK(KP))
IF (NFLAG) 50,50,40
40 DAT=DEL1*DEL2*SQRT(DL(KP)**2+DM(KP)**2+DN(KP)**2)/DK(KP)
DO 45 J=1,NPP
IF (QQ(J)) 45,45,42
42 DAS=DELSAS*SQRT(DLP(J)**2+DMP(J)**2+DNP(J)**2)/DKP(J)
VA=VA+DAT*DAS
45 CONTINUE
IF (KP=NPI) 50,47,47
47 SA=0.0
DO 48 J=1,NPP
SA=SA+SQRT(DLP(J)**2+DMP(J)**2+DNP(J)**2)/DKP(J)
48 CONTINUE
SA=SADELSA
WRITE (6,206)VA,IS,SA
50 CONTINUE
IF (KP=NPI) 6,28,28
6 KP=KP+1
DO 13 J=1,NPP
T(J)=XP(J)-Y(KP)
U(J)=YP(J)-Y(KP)
V(J)=ZP(J)-Z(KP)
R=DL(KP)*T(J)+DM(KP)*U(J)+DN(KP)*V(J)
IF (R) 11,11,9
9 RP= -(DLP(J)*T(J)+DMP(J)*U(J)+DNP(J)*V(J))
IF (RP) 11,11,12
11 QQ(J)=0.0
GO TO 13
12 W= (T(J)*T(J)+J(J)*U(J)+V(J)*V(J))**2
QQ(J)=(R*RP)/W
13 CONTINUE
IF (NOIS) 4,4,16
16 IF (KLM) 17,17,26
17 READ (11      )ID0
IF (ID0) 20,20,19
19 DO 2 I=1,NOIS
IF (MIS(I)=ID0) 2,5,2
2 CONTINUE
GO TO 17
20 IF (KLM) 21,21,22
21 WRITE (6,201)IT,IS
NOIS =0
REWIND 11
GO TO 4

```



TABLE C-1
(CONTINUED)

```

5   BACKSPACE 11                               15681130
     KLM=KLM+1                                15681140
     READ (11) IDD,CA(KLM),CB(KLM),CC(KLM),CD(KLM),CE(KLM),CF(KLM),
1   CG(KLM),CH(KLM),CI(KLM),CJ(KLM),LL(KLM),KNB(KLM),NB,DSSI(KLM),      1568001
2   QLIM1(1),QLIM2(1),QLIM3(1),QLIM4(1),GRIDS(1),(BA(KLM,I),BR(KLM,I)) 1568002
3   BC(KLM,I),BD(KLM,I),BE(KLM,I),BF(KLM,I),BG(KLM,I),BH(KLM,I),      1568003
4   BI(KLM,I),BJ(KLM,I),DBSI(KLM,I),I=1,NB)                            1568005
     IF (KLM=NOIS) 17,26,22                           15681200
22  IF (KLM=NOIS) 25,26,25                           15681210
25  WRITE (6,202)KLM,NOIS,IT,IS                  15681220
26  DO 30 I=1,KLM                                15681230
     A=CA(I)                                     15681240
     B=CB(I)                                     15681250
     C=CC(I)                                     15681260
     D=CD(I)                                     15681270
     E=CE(I)                                     15681280
     F=CF(I)                                     15681290
     G=CG(I)                                     15681300
     H=CH(I)                                     15681310
     QI=CI(I)                                     15681320
     QJ=CJ(I)                                     15681330
     NSIGMA=I                                    15681340
     NBDD=KNB(I)                                 15681350
     CALL MTEST                                  15681360
30  CONTINUE                                     15681370
     GO TO 4                                     15681380
28  RETURN                                       15681390
     END
$IBFTC GRID LIST,REF,DECK,M94,XR7
SUBROUTINE GRID                               15680020
DIMENSION CA(10 ),CB(10 ),CC(10 ),CD(10 ),CE(10 ),CF(10 ),CG(10 ), 15680030
1  CH(10 ),CI(10 ),CJ(10 ),BA(10 ,6),BB(10 ,6),BC(10 ,6),BD(10 ,6), 15680040
2  BE(10 ,6),BF(10 ,6),BG(10 ,6),BH(10 ,6),BI(10 ,6),BJ(10 ,6),      15680050
3  X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),DMP(875), 15680060
4  DNP(875),LL(10 ),KNB(10 ),DSSI(10 ),QLIM1(10 ),QLIM2(10 ),       15680070
5  QLIM3(10 ),QLIM4(10 ),GRIDS(10 ),DBSI(10 ,6)                         15680080
DIMENSION DKP(875),DL(875),DM(875),DN(875),DK(875),PHI(875),
1  PSI(875),MIS(10 ),T(875),U(875),V(875),QQ(875),    VPST(875) 15680090
COMMON /ECOM/ CA, CB, CC, CD, CE, CF
1   , CG, CH, CI, CJ, BA, BB
2   , BC, BD, BE, BF, BG, BH
3   , BI, BJ, X, Y, Z, XP
4   , YP, ZP, DLP, DMP, DNP, LL
5   , KNB, DSSI, QLIM1, QLIM2, QLIM3, QLIM4
6   , GRIDS, DBSI, DKP, DL, DM, DN
7   , DK, PHI, PSI, MIS, T, U
8   , V, QQ, VPST, NGO, NDO, NP
9   , IP, IL, IH, IT, IS, NC
1   , NLD, IU, NFT, NSIGMA, NPP, KP
2   , A, B, C, D, E, F
3   , G, H, QI, QJ, RIGHT, DEL1
4   , DEL2, DEL1P, DEL2P, NBDD, NVP, NOIS, NAREA, LITEMS
GO TO (1,2,3,4,7),NGO
1  IL=0                                         15680410
1  IP=IT                                        15680420
1  GO TO 5                                      15680430
2  IL=-1                                        15680440
2  IP=IT                                        15680450
2  GO TO 6                                      15680460
3  IL=1                                         15680470
3  IP=IS                                        15680480
3  GO TO 8                                      15680490

```



TABLE C-1
(CONTINUED)

	GO TO 5	
4	IL=2	15680500
	IP=IS	15680510
5	CALL POINT	15680520
	GO TO 7	15680530
6	NP=NP	15680540
7	KDEP=LL(IP)	15680550
	GO TO (12,13,14),KDEP	15680560
12	NDO=1	15680570
	GO TO 16	15680580
13	NDO=2	15680600
	GO TO 16	15680610
14	NDO=3	15680620
16	CALL DEPVAR	15680630
	NC=NC	15680640
	IF (NC-1) 20,8,24	15680650
20	WRITE (6,200)	15680660
200	FORMAT (34H NC IS LESS THAN ONE WHA HOPPER)	15680670
	CALL EXIT	15680680
	STOP	15680690
8	IL=IL+2	15680700
	GO TO (30,31,32,33,34),IL	15680710
30	NC=6	15680720
	GO TO 23	15680730
31	NC=5	15680740
	GO TO 23	15680750
32	NC=8	15680760
	GO TO 23	15680770
33	NC=7	15680780
	GO TO 23	15680790
34	NFT=2	15680800
23	RIGHT=1.0	15680810
	GO TO 25	15680820
24	RIGHT=0.0	15680830
25	RETURN	15680840
	END	15680850
	SIBFTC POINT LIST,REF,DECK,M94,XR7	
	SUBROUTINE POINT	15680020
	DIMENSION CA(10),CB(10),CC(10),CD(10),CE(10),CF(10),CG(10),	15680030
1	CH(10),CI(10),CJ(10),BA(10 ,6),BR(10 ,6),SC(10 ,6),SD(10 ,6),	15680040
2	SH(10 ,6),SF(10 ,6),SG(10 ,6),SH(10 ,6),SI(10 ,6),SJ(10 ,6),	15680050
3	X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),DMP(875),	15680060
4	DNP(875),LL(10),KNB(10),DSSI(10),QLIM1(10),QLIM2(10),	15680070
5	QLIM3(10),QLIM4(10),GRIDS(10),DBSI(10 ,6)	15680080
	DIMENSION DKP(875),DL(875),DM(875),DK(875),PHI(875),	15680090
1	PSI(875),MIS(10),T(875),U(875),V(875),QQ(875), VPST(875)	15680100
	COMMON /ECONM/ CA, CB, CC, CD, CE, CF	15680130
1	, CG, CH, CI, CJ, BA, SH	15680140
2	, BC, BD, SE, BF, BG, BH	15680150
3	, BI, BJ, X, Y, Z, XP	15680160
4	, YP, ZP, DLP, DMP, DNP, LL	15680170
5	, KNB, DSSI, QLIM1, QLIM2, QLIM3, QLIM4	15680180
6	, GRIDS, DBSI, DKP, DL, DM, DN	15680190
7	, DK, PHI, PSI, MIS, T, U	15680200
8	, V, QQ, VPST, NGO, NDO, NP	15680210
9	, IP, IL, IH, IT, IS, NC	15680220
1	, NLD, IU, NFT, NSIGMA, NPP, KP	15680230
2	, A, B, C, D, E, F	15680240
3	, G, H, QI, QJ, RIGHT, DEL1	15680250
4	, DEL2, DEL1P, DEL2P, NBDD, NVP, NOIS, NAREA, LTEVS	15680260
	I=0	15680410



TABLE C-1
(CONTINUED)

IR=LL(IP)		15680420
IF (GRIDS(IP) .LT. 2.0) GO TO 40		
IF (QLIM2(IP) .GT. QLIM1(IP)) GO TO 1		
STLM2 = QLIM2(IP)		
QLIM2(IP) = QLIM1(IP)		
QLIM1(IP) = STLM2		
1 IF (QLIM4(IP) .GT. QLIM3(IP)) GO TO 3		
STLM2 = QLIM4(IP)		
QLIM4(IP) = QLIM3(IP)		
QLIM3(IP) = STLM2		
3 I=I+1		15680430
GO TO (2,4,6),IR		15680440
2 DELY=(QLIM2(IP)-QLIM1(IP))/GRIDS(IP)		15680450
DELZ=(QLIM4(IP)-QLIM3(IP))/GRIDS(IP)		15680460
QLIM1(IP)=QLIM1(IP)+DELY/2.0		15680470
QLIM3(IP)=QLIM3(IP)+DELZ/2.0		15680480
Y(I)=QLIM1(IP)		15680490
Z(I)=QLIM3(IP)		15680500
DEL1=DELY		15680510
DEL2=DELZ		15680520
8 I=I+1		15680530
Y(I)=Y(I-1)+DELY		15680540
Z(I)=QLIM3(IP)		15680550
QMAX=AMAX1(Y(I),QLIM2(IP))		15680560
IF (QMAX-QLIM2(IP)) 10,8,10		15680570
10 QLIM3(IP)=QLIM3(IP)+DELZ		15680580
QMAX=AMAX1(QLIM3(IP),QLIM4(IP))		15680590
IF (QMAX-QLIM4(IP)) 40,12,40		15680600
12 I=I+1		15680610
Y(I)=QLIM1(IP)		15680620
Z(I)=QLIM3(IP)		15680630
GO TO 8		15680640
4 DELX=(QLIM2(IP)-QLIM1(IP))/GRIDS(IP)		15680650
DELZ=(QLIM4(IP)-QLIM3(IP))/GRIDS(IP)		15680660
QLIM1(IP)=QLIM1(IP)+DELX/2.0		15680670
QLIM3(IP)=QLIM3(IP)+DELZ/2.0		15680680
X(I)=QLIM1(IP)		15680690
Z(I)=QLIM3(IP)		15680700
DEL1=DELX		15680710
DEL2=DELZ		15680720
20 I=I+1		15680730
X(I)=X(I-1)+DELX		15680740
Z(I)=QLIM3(IP)		15680750
QMAX=AMAX1(X(I),QLIM2(IP))		15680760
IF (QMAX-QLIM2(IP)) 28,20,28		15680770
28 QLIM3(IP)=QLIM3(IP)+DELZ		15680780
QMAX=AMAX1(QLIM3(IP),QLIM4(IP))		15680790
IF (QMAX-QLIM4(IP)) 40,22,40		15680800
22 I=I+1		15680810
X(I)=QLIM1(IP)		15680820
Z(I)=QLIM3(IP)		15680830
GO TO 20		15680840
6 DELX=(QLIM2(IP)-QLIM1(IP))/GRIDS(IP)		15680850
DELY=(QLIM4(IP)-QLIM3(IP))/GRIDS(IP)		15680860
QLIM1(IP)=QLIM1(IP)+DELX/2.0		15680870
QLIM3(IP)=QLIM3(IP)+DELY/2.0		15680880
X(I)=QLIM1(IP)		15680890
Y(I)=QLIM3(IP)		15680900
DEL1=DELX		15680910
DEL2=DELY		15680920
30 I=I+1		15680930



TABLE C-1
(CONTINUED)

```

X(I)=X(I-1)+DELX          15680940
Y(I)=QLIM3(IP)            15680950
QMAX=AMAX1(X(I),QLIM2(IP)) 15680960
IF (QMAX-QLIM2(IP)) .38,.30,.38 15680970
38 QLIM3(IP)=QLIM3(IP)+DELY 15680980
QMAX=AMAX1(QLIM3(IP),QLIM4(IP)) 15680990
IF (QMAX-QLIM4(IP)) .40,.32,.40 15681000
32 I=I+1                  15681010
X(I)=QLIM1(IP)            15681020
Y(I)=QLIM3(IP)            15681030
GO TO 30                  15681040
40 NP=I                   15681050
RETURN                     15681060
END
$IBFTC DIRECT LIST,REF,DECK,M94,XR7
SUBROUTINE DIRECT          15680020
DIMENSION CA(10 ),CB(10 ),CC(10 ),CD(10 ),CE(10 ),CF(10 ),CG(10 ), 15680030
1 CH(10 ),CI(10 ),CJ(10 ),BA(10 ,6),BB(10 ,6),BC(10 ,6),BD(10 ,6), 15680040
2 BE(10 ,6),BF(10 ,6),BG(10 ,6),BH(10 ,6),BI(10 ,6),BJ(10 ,6), 15680050
3 X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),DNP(875), 15680060
4 DNP(875),LL(10 ),KNB(10 ),DSSI(10 ),QLIM1(10 ),QLIM2(10 ), 15680070
5 QLIM3(10 ),QLIM4(10 ),GRIDS(10 ),DBSI(10 ,6) 15680080
DIMENSION DKP(875),DL(875),EM(875),DN(875),OK(875),PHI(875), 15680090
1 PSI(875),MIS(10 ),T(875),U(875),V(875),OO(875), VPST(875) 15680100
COMMON /ECOM4/ CA, CB, CC, CD, CE, CF 15680130
1 , CG , CH , CI , CJ , BA , BB 15680140
2 , BC , BD , BE , BF , BG , BH 15680150
3 , BI , BJ , X , Y , Z , XP 15680160
4 , YP , ZP , DLP , DNP , LL 15680170
5 , KNB , DSSI , QLIM1 , QLIM2 , QLIM3 , QLIM4 15680180
6 , GRIDS , DBSI , DKP , DL , DM , DN 15680190
7 , OK , PHI , PSI , MIS , T , U 15680200
8 , V , OO , VPST , NSG , NDO , NR 15680210
9 , IP , IL , IH , IT , IS , NC 15680220
1 , NLD , IU , NFT , NSIGMA , NPP , KDP 15680230
2 , A , B , C , D , E , F 15680240
3 , G , H , OI , QJ , RIGHT , DEL1 15680250
4 , DEL2 , DEL1P , DEL2P , NEDD , NVP,NOIS,NAREA,LITEMS 15680260
NLD=NLD                  15680410
IF (NP) 11,11,5           15680420
5 GO TO (1+2,5,2,10,11),NLD 15680430
1 IU=IT                  15680440
LW=0                      15680450
GO TO 3                  15680460
2 IU=IS                  15680470
LW=1                      15680480
3 NSIGMA=0                15680490
DO 4 I=1,NP               15680500
4 DL(I)=(CA(IU)*X(I)+CB(IU)*Y(I)+CE(IU)*Z(I)+CG(IU))*DSSI(IU) 15680510
DM(I)=(CO(IU)*X(I)+CB(IU)*Y(I)+CF(IU)*Z(I)+CH(IU))*DSSI(IU) 15680520
DN(I)=(CE(IU)*X(I)+CF(IU)*Y(I)+CC(IU)*Z(I)+CI(IU))*DSSI(IU) 15680530
4 CONTINUE                 15680540
KDP=LL(IU)                15680550
GO TO (20,21,22),KDP     15680560
20 NFT=1                  15680570
GO TO 25                  15680580
21 NFT=3                  15680590
GO TO 25                  15680600
22 NFT=4                  15680610
25 CALL FTST               15680620
NLD=NLD                  15680630

```



TABLE C-1
(CONTINUED)

```

8      GO TO (1,2,8,9,10,11),NLD          15680640
8      DO 30 I=1,NP                      15680650
     DK(I)=ABS(DL(I))                  15680660
30     CONTINUE                         15680670
     GO TO 11                           15680680
9      DO 31 I=1,NP                      15680690
     DK(I)=ABS(DM(I))                  15680700
31     CONTINUE                         15680710
     GO TO 11                           15680720
10     DO 32 I=1,NP                      15680730
     DK(I)=ABS(DN(I))                  15680740
32     CONTINUE                         15680750
11     RETURN                           15680760
     END                               15680770

SIBFTC DEPVAR LIST,REF,DECK,M94,XR7
SUBROUTINE DEPVAR
DIMENSION CA(10 ),CB(10 ),CC(10 ),CD(10 ),CE(10 ),CF(10 ),CG(10 ),15680030
1 CH(10 ),CI(10 ),CJ(10 ),BA(10 ,6),BB(10 ,6),BC(10 ,6),BD(10 ,6),15680040
2 BE(10 ,6),EF(10 ,6),EG(10 ,6),BH(10 ,6),BI(10 ,6),BJ(10 ,6),15680050
3 X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),DMP(875),15680060
4 DNP(875),LL(10 ),KNB(10 ),DSSI(10 ),QLIM1(10 ),QLIM2(10 ),15680070
5 QLIM3(10 ),QLIM4(10 ),GRIDS(10 ),DBSI(10 ,6)          15680080
DIMENSION DKP(875),DL(875),DM(875),DN(875),PHI(875),15680090
1 PSI(875),MIS(10 ),T(875),U(875),V(875),QQ(875),    VPST(875) 15680100
COMMON /ECOM/ CA, CB, CC, CD, CE, CF
1   , CG, CH, CI, CJ, DA, BB 15680130
2   , BC, BD, BE, BF, BG, BH 15680140
3   , BI, BJ, X, Y, Z, XP 15680150
4   , YP, ZP, DLP, DMP, DNP, LL 15680160
5   , KNB, DSSI, QLIM1, QLIM2, QLIM3, QLIM4 15680170
6   , GRIDS, DBSI, DKP, DL, DM, DN 15680180
7   , DK, PHI, FSI, MIS, T, U 15680190
8   , V, QQ, VPST, NGO, NDO, NP 15680200
9   , IP, IL, IH, IT, IS, NC 15680210
1   , NLD, IU, NFT, NSIGMA, NPP, KP 15680220
2   , A, B, C, D, E, F 15680230
3   , G, H, QI, QJ, RIGHT, DEL1 15680240
4   , DEL2, DEL1P, DEL2P, N8DD, NVP,NOIS,NAREA,LITEMS 15680250
NPP=0
IF (N8DD) 600,600,602 15680260
600 NFLAG=1 15680270
GO TO 603 15680280
602 NFLAG=2 15680290
603 GO TO (1,101,201),NDO 15680300
1 DO 2 I=1,NP 15680310
PSI(I)=CD(IP)*Y(I)+CE(IP)*Z(I)+CG(IP) 15680320
2 CONTINUE 15680330
IF (CA(IP)) 5,3,5 15680340
3 DO 303 I=1,NP 15680350
IF (PSI(I)) 303,14,303 15680360
303 CONTINUE 15680370
DO 304 I=1,NP 15680380
X(I)=(CB(IP)*Y(I)**2+CC(IP)*Z(I)**2+2.0*(CF(IP)*Y(I)*Z(I)+CH(IP)
1 *Y(I)+CI(IP)*Z(I))+CJ(IP))/(2.0*PSI(I)) 15680390
304 CONTINUE 15680400
GO TO 11 15680410
5 J=0 15680420
DO 306 I=1,NP 15680430
TER=(PSI(I)**2-CA(IP)*(CB(IP) *Y(I)**2+CC(IP) *Z(I)**2+CJ(IP)+2.0*15680440
1 *(CF(IP)*Y(I)*Z(I)+CH(IP)*Y(I)+CI(IP)*Z(I))) 15680450
IF (TER) 605,7,8 15680460

```



TABLE C-1
(CONTINUED)

605	GO TO (306,606),NFLAG	15680640
606	WRITE (6,900)Y(I),Z(I),LITEMS	15680650
900	FORMAT (10H POINT Y=E20.8,2X,2HZ=E20.8,20H IS NOT ON SURFACE I3)15680660	15680660
	GO TO 306	15680670
8	PHI(I)=SCRT(TER)	15680680
	GO TO 9	15680690
7	X(I)=-PSI(I)/CA(IP)	15680700
	GO TO 10	15680710
9	X(I)=(PHI(I)-PSI(I))/CA(IP)	15680720
	NPP=NPP+1	15680730
	XP(NPP)=(-PHI(I)-PSI(I))/CA(IP)	15680740
	YP(NPP)=Y(I)	15680750
	ZP(NPP)=Z(I)	15680760
10	J=J+1	15680770
	X(J)=X(I)	15680780
	Y(J)=Y(I)	15680790
	Z(J)=Z(I)	15680800
306	CONTINUE	15680810
	NP=J	15680820
	IF (NPP) 402,402,400	15680830
400	DO 401 I=1,NPP	15680840
	NP=NP+1	15680850
	X(NP)=XP(I)	15680860
	Y(NP)=YP(I)	15680870
	Z(NP)=ZP(I)	15680880
401	CONTINUE	15680890
402	NP=NP	15680900
11	J=NP	15680910
	NOB=KNE(IP)	15680920
	IF (NOB) 13,13,4	15680930
4	DO 308 I=1,NOB	15680940
	NP=J	15680950
	J=0	15680960
	DO 308 JQ=1,NP	15680970
	THETA=(BA(IP,I)*X(JQ)**2+BB(IP,I)*Y(JQ)**2+BC(IP,I)*Z(JQ)**2	15680980
	1 +BJ(IP,I)+2.0*((ED(IP,I)*Y(JQ)+ED(IP,I)*Z(JQ)+SG(IP,I))*X(JQ)	15680990
	2 +(EF(IP,I)*Z(JQ)+EH(IP,I))*Y(JQ)+EI(IP,I)*Z(JQ)))*DRSI(IP,I)	15681000
	IF (THETA) 608,608,320	15681010
608	GO TO (308,610),NFLAG	15681020
610	WRITE (6,902)X(JQ),Y(JQ),Z(JQ),LITEMS	15681030
902	FORMAT (10H POINT X=E20.8,2X,2HZ=E20.8,2X,2HZ=E20.8,20H IS NOT ON SURFACE I3)	15681040
	IN SURFACE I3)	15681050
	GO TO 308	15681060
320	J=J+1	15681070
	X(J)=X(JQ)	15681080
	Y(J)=Y(JQ)	15681090
	Z(J)=Z(JQ)	15681100
308	CONTINUE	15681110
	NP=J	15681120
13	IF (NP) 612,612,611	15681130
612	WRITE (6,904) LITEMS	15681140
904	FORMAT (25H NO POINTS ON SURFACE I3)	15681150
611	NC=1	15681160
12	RETURN	15681170
14	I=I	15681180
	IL=IL+2	15681190
	GO TO (19,18,15,16,17),IL	15681200
15	WRITE (6,500)LITEMS	15681210
500	FORMAT (12H SA I3,5H= NOP)	15681220
	NC=4	15681230
	GO TO 12	15681240



TABLE C-1
(CONTINUED)

```

16      WRITE (6,502)IP,IT,IP,IT,IS,IP          15681250
502    FORMAT (12H           VS I3,2X,I3,4H ,VP I3,2X,I3,6H K, VS I3,2X,
1   I3,10H = NOP - P )
      GO TO 20
17      WRITE (6,504)IP,IT,IP,IT,IS,IP,IS,IP,I 15681260
504    FORMAT (10H           VS I3,2X,I3,5H , VP I3,2X,I3, 6H K, VS I3,2X,
1   I3,5H , VP I3,2X,I3,2X,I3,14H = NOL - P - Q )
      GO TO 20
18      WRITE (6,506)IS,IP,IP,IT,IP,IT          15681270
506    FORMAT (12H           VS I3,2X,I3,5H , VS I3,2X,I3,5H , VP I3,2X,
1   I3,14H K , = NOP - P )
      NC=2
      GO TO 12
19      WRITE (6,508)IS,IP,I          15681280
508    FORMAT (12H           VP I3,2X,I3,2X,I3,12H = NOP - P,O )
      NC=3
      GO TO 12
20      IF (IH) 510,511,511          15681290
510    NC=6
      GO TO 12
511    NC=5
      GO TO 12
101     DO 22 I=1,NP          15681300
      PSI(I)=CD(IP)*X(I)+CF(IP)*Z(I)+CH(IP)
22      CONTINUE          15681310
102     IF (CB(IP)) 105,103,105          15681320
103     DO 23 I=1,NP          15681330
      IF (PSI(I)) 23,14,23          15681340
23      CONTINUE          15681350
104     DO 24 I=1,NP          15681360
      Y(I)=-(CA(IP)*X(I)**2+CC(IP)*Z(I)**2+CJ(IP)+2.0*(CE(IP)*X(I)*Z(I)
1   +CG(IP)*X(I)+CI(IP)*Z(I)))/(2.0*PSI(I))          15681370
24      CONTINUE          15681380
      GO TO 11
105     J=0
      DO 26 I=1,NP          15681390
      TER=(PSI(I)**2-CB(IP)*(CA(IP)*X(I)**2+CC(IP)*Z(I)**2+CJ(IP)+2.0*
1   (CE(IP)*X(I)*Z(I)+CG(IP)*X(I)+CI(IP)*Z(I))))          15681400
1   IF (TER) 620,107,108          15681410
620     GO TO (26,622),NFLAG          15681420
622     WRITE (6,910)X(I),Z(I),LTFMS          15681430
910     FORMAT (10H POINT X=E20.8,2X,2HZ=E20.8,20H IS NOT ON SURFACE I3)          15681440
      GO TO 26
108     PHI(I)=SQRT(TER)          15681450
      GO TO 109
107     Y(I)=-PSI(I)/CB(IP)          15681460
      GO TO 25
109     Y(I)=(PHI(I)-PSI(I))/CB(IP)          15681470
      NPP=NPP+1
      XP(NPP)=X(I)
      YP(NPP)=(-PHI(I)-PSI(I))/CB(IP)
      ZP(NPP)=Z(I)
25      J=J+1
      X(J)=X(I)
      Y(J)=Y(I)
      Z(J)=Z(I)
26      CONTINUE          15681480
      NP=J
      IF (NPP) 406,406,404          15681490
404     DO 405 I=1,NPP          15681500
      NP=NP+1

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TABLE C-1
(CONTINUED)

X(NP)=XP(I)	15681860
Y(NP)=YP(I)	15681870
Z(NP)=ZP(I)	15681880
405 CONTINUE	15681890
406 NP=NP	15681900
GO TO 11	15681910
201 DO 30 I=1,NP	15681920
PSI(I)=CE(IP)*X(I)+CF(IP)*Y(I)+CI(IP)	15681930
30 CONTINUE	15681940
202 IF (CC(IP)) 205,203,205	15681950
203 DO 32 I=1,NP	15681960
IF (PSI(I)) 32,14,32	15681970
32 CONTINUE	15681980
204 DO 34 I=1,NP	15681990
Z(I)=-((CA(IP)*X(I)**2+CB(IP)*Y(I)**2+CJ(IP)+2.0*(CD(IP)*X(I)*Y(I))	15682000
+CG(IP)*X(I)+CH(IP)*Y(I)))/(2.0*PSI(I))	15682010
34 CONTINUE	15682020
GO TO 11	15682030
205 J=0	15682040
DO 36 I=1,NP	15682050
TER=(PSI(I)**2-CC(IP)*(CA(IP)*X(I)**2+CB(IP)*Y(I)**2+CJ(IP)+2.0*	15682060
1 *(CD(IP)*X(I)*Y(I)+CG(IP)*X(I)+CH(IP)*Y(I)))	15682070
IF (TER) 630,207,208	15682080
630 GO TO (36,632),NFLAG	15682090
632 WRITE (6,920) X(I),Y(I),LITEMS	15682100
920 FORMAT (10H POINT X=E20.8,2X,2HY=E20.8,20H IS NOT ON SURFACE I3)	15682110
GO TO 36	
208 PHI(I)=SQRT(TER)	15682120
GO TO 209	15682130
207 Z(I)=-PSI(I)/CC(IP)	15682140
GO TO 35	15682150
209 Z(I)=(PHI(I)-PSI(I))/CC(IP)	15682160
NPP=NPP+1	15682170
XP(NPP)=X(I)	15682180
YP(NPP)=Y(I)	15682190
ZP(NPP)=(-PHI(I)-PSI(I))/CC(IP)	15682200
35 J=J+1	15682210
X(J)=X(I)	15682220
Y(J)=Y(I)	15682230
Z(J)=Z(I)	15682240
36 CONTINUE	15682250
NP=J	15682260
IF (NPP) 412,412,408	15682270
408 DO 409 I=1,NPP	15682280
NP=NP+1	15682290
X(NP)=XP(I)	15682300
Y(NP)=YP(I)	15682310
Z(NP)=ZP(I)	15682320
409 CONTINUE	15682330
412 NP=NP	15682340
GO TO 11	15682350
FND	15682360
\$IBFTC FTTEST LIST,REF,DECK,M94,XR7	15682370
SUBROUTINE FTTEST	
DIMENSION CA(10),CB(10),CC(10),CD(10),CE(10),CF(10),CG(10),	15680020
1 CH(10),CI(10),CJ(10),BA(10 ,6),BB(10 ,6),BC(10 ,6),BD(10 ,6),	15680030
2 BE(10 ,6),BF(10 ,6),BG(10 ,6),BH(10 ,6),BI(10 ,6),BJ(10 ,6),	15680040
3 X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),DMP(875),	15680050
4 DNP(875),LL(10),KNB(10),DSS(10),QLIM1(10),QLIM2(10),	15680060
5 QLIM3(10),QLIM4(10),GRIDS(10),DPSI(10 ,6)	15680070
DIMENSION DKP(875),DL(875),DM(875),DN(875),DK(875),PHI(875),	15680080
	15680090



TABLE C-1
(CONTINUED)

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1 PSI(875),MIS(10),T(875),U(875),V(875),QQ(875),
COMMON /ECONM/ CA, CB, CC, CD, CE, CF
2 , CG, CH, CI, CJ, BA, BB
3 , BC, BD, BE, BF, BG, BH
4 , BI, BJ, X, Y, Z, XP
5 , YP, ZP, DLP, DMP, DNP, LL
6 , KNS, DSSI, QLIM1, QLIM2, QLIM3, QLIM4
7 , GRIDS, DBSI, DKP, DL, DM, DN
8 , DK, PHI, PSI, MIS, T, U
9 , V, QQ, VPST, NGO, NDO, NP
1 , IP, IL, IH, IT, IS, NC
1 , NLD, IU, NFT, NSIGMA, NPP, KP
2 , A, B, C, D, E, F
3 , G, H, QI, QJ, RIGHT, DEL1
4 , DEL2, DEL1P, DEL2P, MBDO, NVP, NOIS, NAREA, LITEMS
NFT=NFT
IF (NP) 2,2,4
2 NLD=6
GO TO 15
4 GO TO (1,6,101,201),NFT
1 J=0
DO 12 I=1,NP
IF (DL(I)) 10,11,10
11 WRITE (6,402)LITEMS,X(I),Y(I),Z(I)
GO TO 12
10 J=J+1
X(J)=X(I)
Y(J)=Y(I)
Z(J)=Z(I)
DL(J)=DL(I)
DM(J)=DM(I)
DN(J)=DN(I)
12 CONTINUE
NP=J
NLD=3
15 RETURN
6 WRITE (6,402)
400 FORMAT (62H NFT EQUALS TWO WHICH SHOULD NOT BE UNTIL PROGRAM IS
1REVISED )
402 FORMAT (11H SURFACE I3,19H IS VERTICAL AT X=E20.8,2X,2HY=E20.8,15680630
1 2X,2HZ=E20.8 )
CALL EXIT
STOP
101 J=0
DO 14 I=1,NP
IF (DM(I)) 110,13,110
13 WRITE (6,402)LITEMS,X(I),Y(I),Z(I)
GO TO 14
110 J=J+1
X(J)=X(I)
Y(J)=Y(I)
Z(J)=Z(I)
DL(J)=DL(I)
DM(J)=DM(I)
DN(J)=DN(I)
14 CONTINUE
NP=J
NLD=4
GO TO 15
201 J=0
DO 202 I=1,NP

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TABLE C-1
(CONTINUED)

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203 IF (DN(I)) 210,203,210          15680870
    WRITE (6,402)LITEMS,X(I),Y(I),Z(I) 15680880
    GO TO 202                         15680890
210 J=J+1                           15680900
    X(J)=X(I)                         15680910
    Y(J)=Y(I)                         15680920
    Z(J)=Z(I)                         15680930
    DL(J)=DL(I)                       15680940
    DM(J)=DM(I)                       15680950
    DN(J)=DN(I)                       15680960
202 CONTINUE                         15680970
    NP=J                             15680980
    NLD=5                            15680990
    GO TO 15                          15681000
    END
S18FTC MTEST L1ST,REF,DECK,M94,XR7
SUBROUTINE MTEST                      15680020
DIMENSION CA(10 ),CB(10 ),CC(10 ),CE(10 ),CF(10 ),CG(10 ),15680030
1 CH(10 ),CI(10 ),CJ(10 ),RA(10 ,6),BB(10 ,6),BC(10 ,6),BP(10 ,6),15680040
2 BE(10 ,6),BF(10 ,6),BG(10 ,6),BI(10 ,6),BJ(10 ,6),15680050
3 X(875),Y(875),Z(875),XP(875),YP(875),ZP(875),DLP(875),QMP(875),15680060
4 DNP(875),LL(10 ),KNB(10 ),DSSI(10 ),QLIM1(10 ),QLIM2(10 ),15680070
5 QLIM3(10 ),QLIM4(10 ),GRIDS(10 ),RSI(10 ,6) 15680080
DIMENSION DKP(875),DL(875),DN(875),DK(875),PHI(875),15680090
1 PSI(875),MIS(10 ),T(875),U(875),V(875),QQ(875),VPST(875) 15680100
COMMON /ECOM/ CA, CB, CC, CD, CE, CF
1 , CG , CH , CI , CJ , BA , BB 15680130
2 , BC , BD , BE , BF , BG , BH 15680140
3 , BI , BJ , X , Y , Z , XF 15680150
4 , YP , ZP , DLP , DNP , DNP , LI 15680160
5 , KNB , DSSI , QLIM1 , QLIM2 , QLIM3 , QLIM4 15680170
6 , GRIDS , RSI , DKP , DL , DM , DN 15680180
7 , DK , PHI , PSI , MIS , T , U 15680190
8 , V , QQ , VPST , NGO , NDO , NP 15680200
9 , IP , IL , IH , IT , IS , NC 15680210
1 , NLD , IU , NFT , NSIGMA , NPP , KP 15680220
2 , A , B , C , D , E , F 15680230
3 , G , H , OI , OJ , RIGHT , DELL 15680240
4 , DEL2 , DEL1P , DEL2P , NEED , NVP,NSIS,NVCA,LITEMS 15680250
    KP=KP
    GAMMA=A*X(KP)**2+B*Y(KP)**2+C*Z(KP)**2+OJ+2.0*((D*Y(KP)+E*Z(KP)+15680260
1 G)*X(KP)+(F*Z(KP)+H)*Y(KP)+I)*Z(KP)) 15680270
    BET1=A*X(KP)+D*Y(KP)+E*Z(KP)+G 15680280
    BET2=B*Y(KP)+D*X(KP)+F*Z(KP)+H 15680290
    BET3=C*Z(KP)+E*X(KP)+F*Y(KP)+I 15680300
    DO 24 J=1,NPP 15680310
    ALPHA=(A*T(J)+2.0*(B*U(J)+E*V(J)))*T(J)+(B*U(J)+2.0*F*V(J))*U(J) 15680320
1 + C*V(J)**2 15680330
    BETHA=SET1*T(J)+BET2*U(J)+BET3*V(J) 15680340
    IF (ALPHA) 7,5,7 15680350
5 THETAD=-1.0 15680360
    THETAC=-GAMMA/(2.0*BETHA) 15680370
    GO TO 13 15680380
7 TEST=BETHA**2-ALPHA*GAMMA 15680390
    IF (TEST) 8,9,11 15680400
8 GO TO 24 15680410
9 THETAD=-1.0 15680420
    THETAC=-BETHA/ALPHA 15680430
    GO TO 13 15680440
11 THETAC=(-BETHA+SQRT(BETHA**2-ALPHA*GAMMA))/ALPHA 15680450
12 THETAD=(-BETHA-SQRT(BETHA**2-ALPHA*GAMMA))/ALPHA 15680460

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TABLE C-1
(CONTINUED)

13	IF (THETAC) 18,18,10	15680630
10	IF (THETAC-1.0) 14,18,18	15680640
14	XC=X(KP)+THETAC* T(J)	15680650
	YC=Y(KP)+THETAC* U(J)	15680660
	ZC=Z(KP)+THETAC* V(J)	15680670
17	NN=NSIGMA	15680680
	IF (NBDD) 26,26,3	15680690
3	DO 30 I=1,NBDD	15680700
	VAL=(BA(NN,I)*XC**2+BB(NN,I)*YC**2+BC(NN,I)*ZC**2+BJ(NN,I)+2.0*	15680710
1	((BD(NN,I)*YC+BE(NN,I)*ZC+BG(NN,I))*XC+(BF(NN,I)*ZC+BH(NN,I))*	15680720
2	YC+BI(NN,I)*ZC))*DPSI(NN,I)	15680730
	IF (VAL) 30,30,26	15680740
30	CONTINUE	15680750
18	IF (THETAD) 24,24,15	15680760
15	IF (THETAD-1.0) 19,24,24	15680770
19	XD=X(KP)+THETAD* T(J)	15680780
	YD=Y(KP)+THETAD* U(J)	15680790
	ZD=Z(KP)+THETAD* V(J)	15680800
22	NN=NSIGMA	15680810
	IF (NBDD) 26,26,4	15680820
4	DO 34 I=1,NBDD	15680830
	VAL=(BA(NN,I)*XD**2+BB(NN,I)*YD**2+BC(NN,I)*ZD**2+BJ(NN,I)+2.0*	15680840
1	((BD(NN,I)*YD+BF(NN,I)*ZD+BG(NN,I))*XD+(BF(NN,I)*ZD+BH(NN,I))*	15680850
2	YD+BI(NN,I)*ZD))*DPSI(NN,I)	15680860
	IF (VAL) 34,34,26	15680870
34	CONTINUE	15680880
	GO TO 24	15680890
26	QQ(J)=0.0	15680900
24	CONTINUE	15680910
	RETURN	15680920
	END	15680930

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